TEACHING PHILOSOPHY

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1. INTRODUCTION

I interpret the word “teaching” in the broadest sense: it is the act of communicating mathematics. In this sense, I have “taught” on a wide variety of scales: in individual advising relationships, in the classroom at every level of mathematics class, in organizing a monthly program for local high school students, and by authoring textbooks that have worldwide distribution. In the sections that follow I will discuss each of these.

2. TEACHING THE COLLEGE: CLASSROOM TEACHING

I believe that students only learn mathematics when they are actively intellectually engaged. But only the most mathematically mature student can sit in a lecture and be engaged at such a level. Hence, lecture is an appropriate method for the teaching of mathematics in upper level courses and advanced seminars.

Lower level courses are quite different. Rarely does one encounter a precalculus or calculus student who is mature enough to really get something out of a lecture. At best the lectures serve to pace and motivate the student as they work through the assigned homework problems in the text. Perhaps in a class with 100 students that’s about all you can do. But for this reason, I try to lecture as little as possible in lower level classes.

Instead of lecturing I have prepared a set of carefully planned worksheets for precalculus through the first year of calculus. This method of teaching is greatly influenced by my experiences as a graduate student TA in Uri Triesmann’s Emerging Scholars Program. The idea is to utilize the socratic method as much as possible. But appropriate social groups are also important to the learning of mathematics, so I insist that the students spend the class time working through my worksheets in groups. Outside of class my time as professor is spent writing/revising worksheets, and working with individual students who are having trouble. In the classroom, I circulate around the groups to make sure everyone is on the right track, and to give carefully selected hints where appropriate. I also reassign the groups each day to preserve some balance between them.

Worksheet problems are chosen so that they develop the material I am trying to teach. The difficulty level I strive for with these problems is something that is too hard for any one student, and yet is accessible to a group of students working well together. This forces the groups to have high level mathematical discussions, an important part of the learning process. Some of the more difficult worksheet problems are:

(1) Find a finite expression for the number \[ \sqrt{3 + \sqrt{3 + \sqrt{3 + \ldots}}} \]
(2) Construct a function such that $f(f(x)) \neq x$, but $f(f(f(x))) = x$.

(3)

$$f(x) = \begin{cases} 
  x & \text{if } x \text{ is not rational} \\
  0 & \text{if } x \text{ is rational}
\end{cases}$$

\[ \lim_{x \to 0} f(x) = ? \]

(4) Find functions which are defined for every value of $x$, but are
(a) only continuous for a single value of $x$.
(b) not continuous for any value of $x$.
(c) continuous only at those values of $x$ that are not integers.
(d) continuous only at those values of $x$ that are integers.
(e) continuous on every irrational number, but not continuous on any rational number.

(5) Find a function that is differentiable at exactly one point.

Very few students are able to solve these problems on their own. But with enough (very selective!) hints they usually get it. More importantly, these questions serve as group discussion points. My experience has been that these discussions are invaluable in the learning process.

In intermediate level classes where the students have a little more mathematical maturity I think one can expect them to shift from group work to more individual work. When I taught both set theory and point-set topology, I used a modified “Moore method,” where students worked completely individually, and classtime was spent with students presenting their work, and critiquing the work of their peers.

I view the Moore method as quite dangerous. Many students have a lot of trouble adjusting to this kind of instruction. When I was an undergraduate this was the style of my point-set topology class. It was a two-semester sequence. In the first semester there were about 20 enrolled, and in the second there were only two. To address this issue I spend a lot of time motivating students and creating a “friendly” environment. The last time I taught the sequence there were about 15 in the first class, and something like 12 in the sequel.

Most other classes I teach with a more traditional lecture style. Besides the standard precalc-Calc sequence I have taught a wide variety of both upper and lower level classes. For example, I regularly teach differential geometry, and have several times taught a graduate level class in graph theory. I have also developed several lower level topics classes which have the sole purpose of attracting students that would not ordinarily be interested in a mathematics class, while at the same time exposing them to non-trivial mathematics. These include The Mathematics of Poker, Cartography, and Pencil-and-Paper Games.

3. Teaching the region: Claremont Math Circle

I am currently the faculty director for the Claremont Gateway to Exploring the Mathematical Sciences program, our local Math Circle. One Saturday morning each month we bring in approximately 70 local middle school students and provide them with some exposure to things mathematicians are interested in, in a fun and exciting way.

Although the program is open to anyone, we target the local schools that are in low-income, under performing areas. It is exciting to see children from these schools get genuinely excited about mathematics. Equally important, we have a regular attendance of about 10 teachers
from these schools. We believe that through them we are having an indirect effect on many more students.

Our Saturday sessions begin with a 45-60 minute presentation by a local mathematician. This is followed by a one hour activity that is related to the presentation. This combination affords students a glimpse of what mathematician really do, in a very hands-on way.

For more information about this program, including a list of talks for the 2009-10 school year, I encourage you to visit our website: http://ccms.claremont.edu/GEMS

4. Teaching the world: Mathematical exposition

I believe that one of the most important steps in teaching a class on a new topic is to organize the material before the semester begins in a way that makes sense to you. Ideally, you would then go find a textbook where the material is organized in as close of a way as possible to this.

There have been several times in the past where I have been given the task of teaching particular material, and after doing a literature survey decided that there was no book out there that does things they way I would. Sometimes when this happens I just cut and paste different books together to form a coherent text for the class. But occasionally this approach just doesn’t work, so I decide I should write my own lecture notes to serve as the text.

The first time this happened was when I taught multivariable/vector calculus for several consecutive semesters. I was frustrated by the fact that the students come away from these classes not realizing that the basic integral theorems of vector calculus (Green’s, Stokes, Gauss’, etc.) are all manifestations of the same thing. And even worse, these integral theorems are so complex that even the best students don’t remember them just one semester later. The theory of differential forms provides an alternative, as it replaces all of these theorems with the much simpler Generalized Stokes’ theorem. Furthermore, despite popular opinion that differential forms is too sophisticated for this level of student, I was skeptical that this was the case.

So I set about the task of making the theory of differential forms accessible to a sophomore undergraduate student, as an alternate to a traditional vector calculus class. This began as a set of lecture notes for various classes that I taught on the subject. As these notes became more streamlined, Birkhäuser agreed to publish them. The resulting text, A Geometric Approach to Differential Forms was released and distributed worldwide in August of 2006.

The book has done so well that in just three years Birkhäuser contacted me again and asked me if I would like to do a second edition. Coincidentally, I had just finished teaching a class on differential geometry. This class used my differential forms book for the first half, and I had typed up 80 pages of lecture notes for the second. These additional lecture notes have now been fleshed out and incorporated into the new edition of my Differential Forms book, which will come out some time in 2010.

The approach the differential geometry section of my book takes is somewhat unique. There are two types of one-semester differential geometry courses that are generally taught at universities. The first is for undergraduates, and generally covers the classical topics of curves and surfaces in $\mathbb{R}^3$. This is usually done with language and notation that is considered antiquated by modern geometers. Graduate courses in differential geometry focus
on studying abstract manifolds by using the modern machinery of connections. One difficulty often faced by students trying to learn this material is getting used to complicated index notation invented to deal with arbitrarily high dimensional spaces.

In my book I bridge the gap between these two types of classes. This is done by employing the machinery of differential forms to study the “classical” geometry of curves and surfaces in $\mathbb{R}^3$ using the modern language of connections. Since the dimensions are kept low, I avoid the complicated index notation and can still draw pictures. In this way I present an introduction to modern differential geometry in a way that is suitable for someone who has had no prior exposure to the subject.

In addition to the Differential Forms project, I have been in contact with several other publishers about other possible texts. I have a unique, classroom tested approach to linear algebra based on dynamical systems that Birkhäuser is interested in publishing. I have also had many conversations with both the AMS and Springer-Verlag about a text on the “Mathematics of Poker,” based on notes from some classes I have taught on the topic.

5. **One-on-one teaching: Advising**

There are few things more pleasurable as a teacher than individually fostering a talented student. Unfortunately, I have had the honor of doing this only a handful of times. As I am currently at an undergraduate institution, I can only formally advise undergraduates. But the level of the students at Harvey Mudd college is generally above the level of most first (and even second) year graduate students that I have known. I have had the pleasure of advising three Harvey Mudd students. Also, I have served as an informal advisor to several graduate students at other schools, and a few young post-docs.

Two of the Harvey Mudd students I advised are now in top graduate programs. The third is a current student of mine, who is doing an independent exploration in topics in General Relativity.

One of the most pleasurable informal advising arrangements I have experienced is with Ryan Derby-Talbot, who is a former student of my advisor, Cameron Gordon. Ryan came to Pomona college for a one-semester lectureship as a graduate student, which is where I met him. We immediately began meeting weekly. In these meetings I advised him about his thesis, and taught him more about our field. Eventually our conversations turned to new research, and we began work on a project which turned into our first joint publication. Before Ryan finished graduate school we published a second paper, and we currently have one more submitted paper, and another one in its final stages.

I have noticed that there seem to be two types of relationships with graduate advisors. Some people that I have talked to were handed a thesis problem by their advisor, and then were walked through the proof. Others are left to find their own problems and come up with all proofs on their own. I don’t think there is necessarily one right approach. Ideally, everyone would be in the latter camp. Indeed, anyone who aspires to be a research mathematician should have the experience coming up with, and solving, their own problem. On the other hand, not everyone who pursues a PhD in mathematics is destined to be a research mathematician. These students may need some additional guidance.