

Probability Rules

1. A medical laboratory received a shipment of 16 microscopes of which (unbeknownst to the lab) 4 were defective. If a random sample of 3 were taken without replacement, what would be the chances of finding 0 defective units? 1 defective unit? 2 defective units? 3 defective units?

0.3928, 0.4714, 0.1286, 0.0071

2. A survey of Pitzer students finds that 90% like ice cream, 50% like buffalo wings, and 45% like both ice cream and buffalo wings. Are the tastes “liking ice cream” and “liking buffalo wings” independent for Pitzer students? Explain.

If A and B are independent, $P(A \cap B) = P(A)P(B)$

3. At one time it was assumed that Hillary Clinton would win the Democratic party nomination in 2008, that Rudolph Giuliani has a 70% chance of winning the Republican party nomination, and that John McCain has a 30% chance. The political pundits told us that if Hillary Clinton ran against Rudolph Giuliani, she had a 50% chance of winning the election and if she ran against John McCain, she had a 75% chance of winning the election. If after the election you are told that Hillary Clinton was elected, what are the chances that she ran against Rudolph Giuliani?

0.6087

4. Four women and five men apply for three job openings. The manager finds them all “equally qualified” and decides to “randomly draw names out of a hat.” She hires three men and the women seek legal action. The judge asks you to determine the probability of this hiring having truly occurred by random chance.

0.1190

5. The law firm of Dewey, Chetum & Howe in Harvard Square has a dismal record. They lost 5 of their 7 cases last week. “But,” they explain, “nobody’s perfect. We know from past experience that there’s only a 70% chance that we will lose any particular case, and the outcome of each case is independent of the outcome of all the other cases. We just had an extremely unlucky week.” Next week they are scheduled to try 7 cases again. Assume that the law firm’s statements about their probabilities are accurate.

a. What is the probability that this law firm will have an equally dismal record next week (i.e. they lose 5 out of 7 cases)?

b. What is the probability that they will do worse next week than they did last week?

c. What is the probability that they will lose the first 5 cases next week and win the last two?

0.3176, 0.3293, 0.0151

6. There are a total of 7225 thousand persons with multiple jobs in the United States. Of them, 3110 thousand are female, 1742 thousand are single, and 837 thousand are female and single. What is the probability that a randomly selected person with multiple jobs is female and single? What is the probability that a randomly selected person is female or single?

0.1158, 0.5557

7. In a promotion for United Airlines, customers and potential customers were given vouchers. A $\frac{1}{325}$ proportion of these vouchers were for a free round-trip ticket anywhere United flies. How many vouchers would an individual need to collect in order to have a 50% chance of winning at least one free trip?

225

8. Can mutually exclusive events be independent? Explain with an example.

No

9. A hamburger chain found that 75% of all customers use mustard, 80% use ketchup, and 65% use both. What is the probability that a particular customer will use at least one of the two? What is the probability that a mustard user will use ketchup?

0.90, 0.8666

10. Pitzer economics majors are in high demand. They can expect to get job offers from 60% of the firms to which they apply. Susie Senior is busy writing her thesis, and only applies to five firms.

a. What is the probability that Susie will get no job offers?

b. What is the probability that Susie will get three or more job offers?

c. What is the probability that Susie will get two rejection letters followed by three job offers?

0.0102, 0.6830, 0.0346

11. In the language of government statistics, the “labor force” includes all civilians over 16 years of age who are working or looking for work. Select a member of the U.S. labor force at random. Let A be the event that the person selected is white, and B be the event that he or she is employed. In 2005 82.1% of the labor force is white. Of the whites in the labor force, 95.4% were employed. Among non-white members of the labor force, 91.0% were employed.

a. Find the probability that the person chosen is an employed white. Also find the probability that an employed nonwhite is chosen. What is the probability $P(B)$ that the person chosen is employed?

b. Find the probability $P(A|B)$ that a randomly selected member of the labor force is white given that he or she is employed.

0.7832, 0.1629, 0.9461

12. Suppose Andy Roddick makes a successful first serve about 67% of the time. If he serves six times, what is the probability that he gets

a. exactly 4 serves in?

b. no more than 4 serves in?

0.3292, 0.6422

13. Consider a lottery in which three numbers will be drawn at random out of the numbers 1,2,3 and 4. A prize of \$1000 will be given to the person who draws the numbers 123 (one hundred and twenty three) and a prize of \$40 will be given to the person who draws any other arrangement of the numbers 1, 2 and 3 (such as 213, 321, and so on).

a. Determine the probability of getting the \$1000 prize if you have one lottery ticket.

b. Determine the probability of getting the \$40 prize if you have one lottery ticket.

0.0417, 0.2083

14. Suppose that sixty percent of Pitzer students major in the social sciences and forty percent major in the humanities. Assume that five percent of social science majors and ten percent of humanities majors graduate with honors.

a. What is the probability that a student (randomly selected) will graduate with honors?

b. If it is known that a student has graduated with honors, what is the probability that he or she majored in the social science?

0.0700, 0.4286

15. Let events A and B be independent. If $P(A) = 0.40$ and $P(B) = 0.50$, what is $P(A \text{ or } B)$, $P(A \text{ and } B)$, and $P(A|B)$?

0.70, 0.20, 0.40

16. A personnel manager knows that 40% of the workers who are hired without a screening test perform satisfactorily on the job. However, among workers who take the test and later do well on the job, 90% pass it, while amongst those who take the test and later do badly on the job, 80% fail it. What is the probability of satisfactory job performance if a randomly chose applicant a) passes the test and b) fails the test?

0.7500, 0.0769

17. Baskin-Robbins ice-cream parlors advertise 31 different flavors of ice cream. They decide to offer a substantial prize to any customer who orders a triple scoop cone with the winning combination of flavors. What is the probability of winning if

a. each flavor must be different and the order of the flavors does not matter?

b. each flavor must be different and the order of the flavors does matter?

c. each flavor need not be different and the order of the flavors does not matter?

d. each flavor need not be different and the order of the flavors does matter?

0.000222, 0.000037078, 0.00018328, 0.000033567

18. Suppose you are dealt a 5-card hand from a deck of 52 cards. What is the probability that you have a straight flush? (A straight flush is a sequence of cards like A,2,3,4,5 or 4,5,6,7,8 or 10,J,Q,K,A where all the cards are of the same suit)

0.00001539

19. In 1986 the tragic space shuttle Challenger explosion came on NASA's 26th mission. What is the probability of 25 successes in 25 missions if the probability of failure on each mission is 1 percent? 4 percent? 10 percent?

0.7778, 0.3604, 0.0718

20. Suppose the bookstore carries 60 copies of a Ray Charles CD, of which 8 CDs are defective. You and your friend have just bought two copies of the Ray Charles CD. What is the probability that both of your copies are defective?

0.01582

21. You are dealt two cards from a deck of 52 cards.

a. What is the probability that both cards are clubs?

b. What is the probability that both cards are of the same suit?

c. What is the probability that the first card is a club and the second is a diamond?

0.0588, 0.2353, 0.0637

22. You need to borrow \$100,000 to attend college. There is a 90% probability that Wells Fargo will loan you \$100,000. There is an 80% probability that Bank of America will loan you \$60,000. There is a 70% probability that Washington Mutual will loan you \$40,000. Assume that all these loan decisions are independent. What is the probability that you will be able to borrow enough money? If Washington Mutual decides to give you the loan of \$40,000, what is the probability of being able to borrow enough money now?

0.9560, 0.9800

23. Two roommates, Elmer and Fudd, try to get dates for Friday night. Elmer has a 30% probability of getting a date. Fudd has a 40% probability of getting a date. One of them ends up getting a date. What is the probability that it was Elmer?

0.3913

24. Professors get numerous calls from students every day. Often students ask for extensions on their problem sets because they are sick, have too much other work, or forgot that the problem set was due. Suppose that the probability of a student requesting an extension is 0.70, and that the probability of a student asking for an extension and being sick is 0.60. Given that a student requests an extension, what is the probability that she is sick?

0.8571

25. It is known that 95% of all personal computers of a particular model will operate for at least 1 year before requiring repair. A manager purchases 20 of these computers. What is the probability that all 20 will work for 1 year before any repair is needed?

0.3585

26. You need to fly to Washington, D.C. for an interview. Unfortunately there is a snow storm on the East Coast. There is a 40% chance that the LAX-Washington, D.C. flights are canceled. There is a 30% chance that the LAX-New York flights will be canceled, and there is a 20% chance that the New York-Washington, D.C. flights will be canceled. What is the probability of being able to get to Washington, D.C. from LAX by plane?

0.8240

27. In the lottery game FANTASY 5 you have to select 5 numbers from the numbers $\{1, 2, 3, \dots, 38, 39\}$ for a jackpot of \$40,000. In this lottery each number must be different and the order of the numbers does not matter.

a. What is the probability of winning the jackpot?

b. What would the probability be if the order of the numbers did matter?

c. What is the probability of not matching any of the 5 numbers?

0.000001736, 0.00000001447, 0.48328

28. Suppose that Siskel gives movies a “thumbs up” 60% of the time, and Siskel and Ebert give movies two “thumbs up” 40% of the time. If Siskel gives a movie a “thumbs up”, what is the probability that Ebert gave the movie a “thumbs up”?

0.6666

29. For the daily lottery game in Illinois, participants select three numbers between 0 and 9. Suppose the numbers cannot be selected more than once, so a winning ticket could be, say, 307.

Also assume that the order of the numbers matters. Purchasing one ticket allows you to select one set of numbers. The winning numbers are announced on TV each night.

- How many different outcomes (three-digit numbers) are possible?
- If you purchase a ticket for a game tonight, what is the likelihood you will win?
- Suppose you purchase three tickets for tonight's drawing and select a different number for each ticket. What is the probability that you will not win with any of the tickets?

720, 0.001388, 0.99583

30. A survey find that 79% of women can check the oil, 58% can change a tire, 52% can change a battery, and 41% can do all of the above. Are all three of these events independent? Explain.

No

31. A student prepares for an exam by studying a list of ten problems. She can solve 6 of them. For the exam, the professor selects 5 questions at random from the list of 10. What is the probability that the student can solve all 5 problems on the exam?

0.0238

32. Mr. Ed has a 10% chance of winning the Kentucky Derby, a 15% chance of winning Preakness, and a 20% chance of winning the Belmont Stakes. What is the probability that he will win at least one of these three horse races?

0.3880

33. When it is raining, the probability that there will be lightening is 28%. When it is not raining, the probability that there will be lightening is only 10%. On average, it rains one day out of the week. If you see lightening, what is the probability that it is raining?

0.31818

34. Suppose that 23% of adults smoke cigarettes. It's known that 57% of smokers and 13% of nonsmokers develop a certain lung condition by the age of 60.

- Explain how these statistics indicate that lung condition and smoking are not independent.
- What's the probability that a randomly selected 60-year old has this lung condition?
- What's the probability that someone with the lung condition was a smoker?

0.2312, 0.5670

35. Bart is a Pitzer freshman. He needs a total of 32 course credits to graduate. The probability that he will get 4 course credits in a semester is 70%. He will never get more than 4 course credits in a semester and refuses to attend summer school. What is the probability that he will graduate in 4 years (8 semesters)?

0.0576

36. Dan's Diner employs three dishwashers. Al washes 40% of the dishes and breaks only 1% of those he handles. Betty and Chuck each wash 30% of the dishes, and Betty breaks only 2% of hers, but Chuck breaks 3% of the dishes he washes. You go to Dan's for supper one night and hear a dish break at the sink. What is the probability that Chuck is on the job?

0.4736

37. The soccer team's shirts have arrived in a big box, and people just start grabbing them, looking for the right size. The box contains 4 medium, 10 large, and 6 extra-large shirts. You want a medium for you and one for your sister. Find the probability of each event described.

- The first two you grab are the wrong size.
- The first medium shirt you find is the third one you check.
- The first four shirts you pick are all extra-large.
- At least one of the first four shirts you check is a medium.

0.63157, 0.14035, 0.0030959, 0.62435

38. A check of dorm rooms on a large college campus revealed that 38% had refrigerators, 52% had TVs, and 21% had both a TV and a refrigerator. What's the probability that a randomly selected dorm room has

- a TV but no refrigerator?
- a TV or a refrigerator, but not both?
- neither a TV nor a refrigerator?

0.310, 0.480, 0.310

39. The following are probabilities of several events in a given sample space: $P(A) = 0.20$, $P(B) = 0.40$, $P(C) = 0.10$, $P(D) = 0.50$, $P(A \cap B) = 0.08$, $P(A \cap D) = 0.00$, $P(C \cup D) = 0.60$.

- Are A and B mutually exclusive? Are they independent?
- How would you characterize the relationship between A and D?
- How would you characterize the relationship between C and D?

independent, mutually exclusive, mutually exclusive

40. Two candidates are running for office in a given city. The probability that the first will be elected is 0.70. The probability that the second will be elected is 0.30. If the first is elected, the probability that a new air terminal will be built is 0.60. If the second is elected, the probability is 0.40. What is the probability that a new air terminal will be built?

0.54

41. A committee is to consist of 5 people, to be chosen from a list of 6 Democrats, 4 Republicans, and 2 Independents.

- How many distinctly constituted committees are possible?
- Assuming an equal chance for each of the 12 candidates to be selected, what is the probability that the committee will have no independents?
- What is the probability that the committee will consist entirely of Democrats?

792, 0.3181, 0.0075757

42. A truck driver has to deliver merchandise from point A to points B, C, D, and E. How many different routes can she take?

24

43. A person owns three securities listed on the New York Stock Exchange. The probability that the price of one of the three securities will go up the next day is $1/5$. The probability that the prices of all three will go up is $1/16$, and the probability that the price of none of them will go up is $5/8$. What is the probability that the price of at least two of the three securities will go up?

0.1125

44. A general can plan a campaign to fight one major battle or three small battles. He believes that he has probability 0.60 of winning the large battle, and probability 0.80 of winning each of the small battles. Victories or defeats in the small battles are independent. The general must win either the large battle or all three small battles to win the campaign. Which strategy should he choose?

large battle

45. The “random walk” theory of securities prices holds that the price movements in disjoint time periods are independent of each other. Suppose that we record only whether the price is up or down each year, and that the probability that our portfolio rises in price in any one year is 0.65. (This probability is approximately correct for a portfolio containing equal dollar amounts of all common stocks listed on the New York Stock Exchange.)

a. What is the probability that our portfolio goes up for three consecutive years?

b. If you know that the portfolio has risen in price 2 years in a row, what probability do you assign to the event that it will go down next year?

c. What is the probability that the portfolio’s value moves in the same direction in both of the next 2 years?

0.2746, 0.35, 0.545

46. In 1998 the Yankees swept the World Series. If you have two evenly matched teams, what is the probability that the seven game series will end after 4 games? 5 games? 6 games? 7 games? Remember that the team who wins 4 games first, wins the series, and no further games are played.

0.1250, 0.250, 0.3125, 0.3125

47. In a large city, 8% of the inhabitants have contracted a particular disease. A test for this disease is positive in 80% of people who have the disease, and is negative in 80% of people who do not have this disease. What is the probability that a person for whom the test result is positive has the disease?

0.25806

48. A life insurance salesman finds that of all the sales he makes, 70% are to people who already own policies. He also finds that of all contacts for which no sale is made, 50% already own life insurance policies. Furthermore, 40% of all contacts result in sales. What is the probability that a sale will be made to a contact who already owns a policy?

0.4827

49. A jury of twelve members is to be selected from a panel consisting of eight men and eight women.

a. How many different jury selections are possible?

b. If the choice is to be made randomly, what is the probability that a majority of the jury members will be men?

1820, 0.2846

50. Show that the probability of the union of the events A and B can be written

$$P(A \cup B) = P(A) + P(B)[1 - P(A|B)]$$

51. An oil company is planning to drill for oil in two locations, Upland and Claremont. Experts estimate the probability of finding oil of acceptable quality to be 20% in Upland, and 25% in Claremont. They also state that if oil of acceptable quality is found in Upland, the probability of finding the same in Claremont is 80%. If oil of acceptable quality is found in Claremont, what is the probability that such oil will be found in Upland?

0.640

52. The probability of A is 30%, and the probability of B is 40%. What is $P(A \cap B)$ if A and B are independent? What is $P(A \cap B)$ if A and B are mutually exclusive? If $P(A \cup B)$ is 70%, what can we say about A and B?

0.120, 0.000, mutually exclusive

53. During the 1960 Presidential election campaign, Richard Nixon visited all 50 states in the U.S. Suppose there is a prize given for picking the last four states he visited. What is the probability of guessing the answer if the order of the visits matters? What if the order of the visits does not matter?

0.0000001809, 0.00000434216

54. Suppose that 30% of people ski. People who ski have a 15% chance of breaking their legs. People who do not ski have a 5% chance of breaking their legs. Suppose you meet someone who has broken their legs. What is the probability that this person is a skier?

0.5625

55. You are a senior applying to law school. There is a 20% chance of getting into Yale, a 25% chance of getting into Stanford, and a 30% chance of getting into Harvard. Assume all these events are independent. What is the probability of getting into at least one of these three law schools?

0.580

56. A stock market analyst claims expertise in picking stocks that will outperform the corresponding industry norms. This analyst is presented with a list of five high technology stocks and a list of five airline stocks, and she is invited to nominate, in order, the three stocks that will do the best on each of these two lists over the next year. The analyst claims that success in just one of these two tasks would be a substantial accomplishment. If, in fact, the choices were made randomly and independently, what would be the probability of success in at least one of the two tasks merely by chance?

0.033

57. A Pitzer student finds that 80% of his college courses have been enjoyable, and the remainder have been boring. This student goes to RateMyProfessor.com and finds that 70% of his enjoyable courses and 25% of his boring courses have been taught by professors who have high ratings. So the next semester the student decides to take four courses, all from professors with high ratings. Assume that this student's reactions to these four courses are all independent of one another. What is the probability that this student will find all four courses to be enjoyable?

0.7103