Economics 91 | Spring 2010
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Probability Distributions

1. Assuming that the conditions of a Bernoulli process are met, determine the following:
a. A test is administered 10 times and the probability of getting the correct answer is 0.90. Use an appropriate formula to figure out the likelihood of getting precisely 9 correct answers.
b. What is the probability of having exactly 3 boys in a family of 10 children (assuming that boys and girls are equally likely and different births are independent events.)
.3874, .1172

2. Small businesses are going bankrupt at a rate of 9.7 per month. What is the probability of at least 3 going bankrupt next month? of exactly 24 businesses going bankrupt?
.9964, .00004755

3. The length life of a computer component is an exponential random variable with a mean of 7 years. If the warranty period if 5 years, what proportion of the components can the manufacturer expect to replace under the warranty? What should the warranty period be if the manufacturer doesn't want to be bothered with having to replace more than 10 percent of the components?
.510, .7375

4. College Board scores are normally distributed with standard deviation of 100 points for both left- and right-handed students. However, the mean math score for left-handed students is 525 whereas it is 500 for right-handed students. In the population as a whole, 10 percent of all students are left-handed. What proportion of right-handed students score over 700? What proportion of left-handed students score over 700? What fraction of students with math scores over 700 are left-handed?
.0228, .0401, .1635

5. In 1991-92 the verbal ability scores on the GRE were approximately normally distributed with a mean of 480 and a standard deviation of 120. Approximately 35% of the students who took this test had scores below what value? Approximately 10% of the students had scores above what value?
433.8, 633.6

6. The weight of medium-sized tomatoes selected at random from a bin at the local supermarket is a random variable with a mean $\mu = 10$ ounces and a standard deviation $\sigma = 2$ ounces. Suppose we pick four tomatoes at random from the bin and put them in a bag. The weight of the bag is a random variable. What is the mean and the standard deviation of the weight of the bag? What is the probability that the bag will weigh less than 38 ounces?
40, 4, .3085

7. Suppose the heights of Americans, measured in inches, has a mean of 68 inches and a variance of 9. What would be the mean and variance of American heights if we measured them in centimeters? Remember that an inch is equal to 2.54 centimeters.
172.72, 58.064

8. Suppose that in the land of Sigma income Y has a mean of 200 and a standard deviation of 50 and that taxes T has a mean of 100 and a standard deviation of 20. If the covariance of Y and T is 500, what is the standard deviation and variance of disposable income Yd = Y – T?
43.59, 1900

9. The following table shows the joint distribution of two random variables X and Y.

	Y	
X	0	5
0	.20	.10
1	.10	.60

a. What is the mean of X? mean of Y?
b. What is the variance of X? variance of Y?
c. What is the covariance of X and Y?
d. What is the correlation coefficient?
0.7, 3.5, 0.21, 5.25, 0.55, 0.5238

10. A food processing company markets a successful line of canned soups. The firm knows the output of its can filling machine is normally distributed and can be programmed to any mean fill setting the company desires. The firm also knows the standard deviation of its production process is 0.05 ounces. What mean fill setting should the firm choose so that 95 percent of the cans its produces will contain *at most* 12 ounces of soup?
11.917

11. Let F be a random variable measuring the Fahrenheit temperature in Nowhere, New Hampshire at noon during the month of March. Exhaustive National Weather Service records indicate that the mean and the variance of F are μ = 25 and $\sigma^2 = 100$. The Celsius temperature is given by $C = \frac{5}{9}(F - 32)$. What is the mean and the standard deviation of the random variable C?
-3.89, 30.86

12. Grove stock has a mean rate of return of 5% and a variance of 64. Motley stock has a mean rate of return of 15% and a variance of 200. The correlation coefficient of the returns on Grove stock and Motley stock is 0.40. Suppose we hold a portfolio which is 40% Grove stock and 60% Motley stock. What is the expected rate of return on the portfolio? What is the variance on the rate of return on the portfolio?
11%, 103.96

13. Biologists and ecologists record the distribution of measurements made on animal species to help study the distribution and evolution of animals. The African finch *Pyrenestes ostrinus* is interesting because the distribution of its bill size has two peaks even though other body measurements follow normal distributions. For example, a study in Cameroon found that the wing length of male finches varies according to a normal distribution with a mean 61.2 mm and standard deviation 1.8 mm.
a. What proportion of male finches have wings longer than 65 mm?
b. What is the wing length that only 2% of male finches exceed?
0.0174, 64.899

14. An investor has \$1,000 to invest and two investment opportunities, each requiring a minimum of \$500. The profit per \$100 from the first can be represented by a random variable X which is equal to \$-5 with probability 0.40 and \$20 with probability 0.60. The profit per \$100 from the second is given by the random variable Y which is equal to \$0 with probability 0.60 and \$25 with probability 0.40. Both X and Y have an expected value of \$10 and are independent. What is the variance in her profit if she invests \$1,000 in the first investment? What is the variance in her profit if she invests \$1,000 in the second investment? What is the variance in her profit if she invests \$500 in the first and \$500 in the second?

15. A market researcher conducts a survey to examine the effectiveness of advertising for a soft drink. She finds that 15% of people both saw the advertising and purchased the soft drink. Also, 45% of all people saw the advertising, and 20% of all people purchased the soft drink. Define a pair of random variables as follows:

$$X = \begin{cases} 1 & \text{saw the advertising} \\ 0 & \text{otherwise} \end{cases}$$

$$Y = \begin{cases} 1 & \text{purchased the soft drink} \\ 0 & \text{otherwise} \end{cases}$$

Find and interpret the covariance between X and Y.

16. a. Why is the Central Limit Theorem important?
b. If $\overline{X}$ has a normal distribution with mean μ and variance σ^2/n, then what random variable has a $N(0,1)$ distribution?
c. If X is distributed $N(0,1)$ and Y is distributed χ^2_n, then what random variable has a Student *t* distribution? How many degrees of freedom does it have?
d. Why does $\dfrac{\overline{X} - \mu}{S_x/\sqrt{n}}$ have a Student *t* distribution?

17. Mensa is an organization whose members possess IQs in the top 2% of the population. To qualify, you must have an IQ greater than 133. If IQs are normally distributed with a mean of 100, what is the standard deviation and variance of IQs?

18. Suppose that your email messages arrive randomly at an average rate of 4 per hour or 1 every 15 minutes.
a. Using the poisson distribution, find the probability that exactly 2 email messages will arrive during a specific 1 hour period.
b. Using the exponential distribution, find the probability that the next email message will not arrive within 30 minutes.

19. If you pay me \$1, I will let you play the following gambling game. You drop your pencil point into the middle of a table of random numbers and look at the digit on which it lands.

If it is a 7, you win \$5.
If it is a 1, you win \$2.

Otherwise, you win nothing.

Find the expected value of the amount you win in one play of this game. Explain in simple language what the expected value tells you about your winnings if you play the game many times. Is it your advantage to pay $1 to play?

20. Weights of newborn children in the United States vary according to the normal distribution with a mean 7.5 pounds and standard deviation of 1.25 pounds. The government classifies a newborn as having low birth weight if the weight is less than 5.5 pounds.

a. What is the probability that a baby chosen at random weighs less than 5.5 pounds at birth?

b. You choose 3 babies at random. What is the probability that their average birth weight is less than 5.5 pounds?

21. About 22% of the residents of California were born outside the United States. You choose a simple random sample of 1,000 California residents for a sample survey on immigration issues. You want to find the probability that 250 or more of the people in your sample were born outside the United States. What is this probability?

22. A community organization plans to ask 100 randomly selected individuals whether they favor a change in the zoning laws. You argue for a sample of 900 individuals instead of 100. You argue that the standard deviation of the proportion $\hat{\pi}$ of the sample who say “yes” will be much smaller. How much smaller will it be?

23. The NASDAQ Composite Index describes the average price of common stock traded over the counter, that is, not on one of the stock exchanges. In 1991 the mean capitalization of the companies in the NASDAQ was $80 million and the median capitalization was $20 million. (A company’s capitalization is the total market value of its stock.) Explain why the mean capitalization is much higher than the median. What does this tell you about the skewness of the distribution?

24. The length of human pregnancies from conception to birth varies according to a distribution that is approximately normal with a mean of 266 days and standard deviation of 16 days.

a. What percent of pregnancies last less than 240 days (that’s about 8 months)?

b. What percent of pregnancies last between 240 and 270 days (roughly between 8 and 9 months)?

c. How long do the longest 20% of pregnancies last?

25. The rate of return on stock indexes (which combine many individual stocks) is approximately normal. Since 1945, the Standard and Poor’s 500 index has had a mean yearly return of about 12%, with a standard deviation of about 16.5%. Take this normal distribution to be the distribution of yearly returns over a long period.

a. In what range do the middle 95% of all yearly returns lie?

b. The market is down for the year if the return on the index is less than zero. In what percent of years is the market down?

c. In what percent of years does the index gain 25% or more?

26. The Gallup Poll once found that about 15% of adults jog. Suppose that in fact the proportion of the adult population who job is $p = 0.15$. What is the probability that the sample proportion of joggers in a simple random sample of size $n = 200$ lies between 14% and 17%? Use a normal approximation.

27. Suppose that 20 year old men have a life expectancy of 78 years with a standard deviation of 9 years. If an insurance company writes policies for 50 men, what is the probability that the average age of death for the policy holders will be less than 77 years? What if the company writes policies for 100 men?

28 Suppose we are concerned with the variability of the number of raisins in a box of raisin bran. Assume that the number of raisins is distributed normally. If a random sample of 16 boxes is taken, what is the probability that the sample variance will be more than twice the population variance?

29. Thelma and Louise play the following game: Thelma starts off with one cent and Louise with two cents. Thelma and Louise simultaneously flip one coin each. If both coins fall on the same side, Louise gives Thelma one cent. If the coins fall on different sides, Thelma gives Louise once cent. The game continues until either Thelma or Louise has all three coins. Write down the probability distribution of X where X is the number of flips required to end the game.

30. An investor is considering two mutual funds, the Slow and Steady (SS) Fund and the High Roller (HR) Fund. Based on past performance, the following probabilities are assigned to the possible percentage rates of return:

SS Fund		HR Fund	
return	probability	return	probability
-10	0.1	-15	0.2
0	0.1	5	0.2
10	0.6	15	0.2
20	0.1	25	0.2
30	0.1	45	0.2

Calculate the means and the standard deviations of these two funds.

31. The correlation coefficient between two random variables X and Y is $\rho = 0.70$. It is given that X = 1, 2 or 3 with equal probability and Y = -1, -2 or -3 with equal probability. Calculate the covariance of X and Y.

32. The accompanying table shows, for credit card holders with one to three credit cards, the joint probabilities for the number of cards owned (X) and the number of credit card purchases made in a week (Y).

# of Cards			# Purchases Y		
X	0	1	2	3	4
1	.08	.13	.09	.06	.03
2	.03	.08	.08	.09	.07
3	.01	.03	.06	.08	.08

Are the number of cards owned and the number of purchases independent? Illustrate or explain why.

33. Let X be the number of between-meal snacks a student eats in a day and let Y be the number of exams she has the next day. Suppose the following is the joint probability distribution of X and Y.

		Y: exams 0	1	2
	0	0.23	0.04	0.03
X: snacks	1	0.12	0.17	0.11
	2	0.05	0.10	0.15

a. What is the expected value of X? of Y?
b. What is the variance of X? of Y?
c. What is the covariance of X and Y?
d. What is the correlation coefficient ρ?
e. Are X and Y independent here? Why?
f. Suppose we define a misery index Z = 3Y - 2X.
    What is the expected value of Z?
    What is the variance of Z?

34. For any two random variables X and Y, what is the relationship between the variance of (X-Y) and the variance of (Y-X)? Why would you expect this?

35. A medical clinic tests blood for a sexually transmitted disease, syphilis, which afflicts 1 out of every 100 persons. Up to now they have been testing each blood sample individually. But since each test is rather expensive, you have suggested that they pool about 50 blood samples together. If the pooled sample tests negative, then they can clear all 50 people with one test. If the pooled sample tests positive, then they have to test all 50 samples individually and run a total of 51 tests.
a. What is the probability that all 50 people in a sample are healthy?
b. What is the probability that at least one person in the sample has syphilis?
c. What is the expected number of tests they will have to run?
d. What is the variance of the number of tests they will have to run?

36. A college basketball player, who sinks 75% of his free throws, comes to the line to shoot a "one and one" (if the first shot is successful, he is allowed a second shot, but no second shot is take if the first is missed; one point is scored for each successful shot). Assume that the outcome of the second shot, if any, is independent of that of the first. Find the expected number of points resulting from the "one and one". Compare this with the expected number of points from a "two-shot foul," where a second shot is allowed irrespective of the outcome of the first.

37. Suppose that 80% of people actually pay their parking tickets, and that the parking tickets are for $30. If campus security issues 15 tickets, how much revenue can they expect to collect? 20 tickets? 25 tickets? What is the probability that they will generate $450 or more if they issue 15 tickets? 20 tickets? 25 tickets?

38. Records indicate that on average, 3.8 breakdowns per day occur on L.A. freeways during the morning rush hour. Assume that the distribution is Poisson.
a. Find the probability that on any given day, there will be fewer than two breakdowns on the freeways during the morning rush hour.
b. Find the probability that on any given day, there will be more than three breakdowns on the freeways during the morning rush hour.

39. Let the random variable Z follow a standard normal distribution with a mean $\mu = 0$ and variance $\sigma^2 = 1$.
a. Find P(Z < 0.75)
b. Find P(Z > -1.25)
c. Find P(-0.33 < Z < 0.66)
d. Find P(-0.80 < Z < -0.25)

40 An investment will be worth $1000, $2000, or $5000 at the end of the year. The probabilities of these values are 0.25, 0.60, and 0.15 respectively. Determine the mean and the variance of the value of this investment.

41.The Bates Motel has 5 rooms. Its experience has been that 20% of the people who make reservations never show up. How many no-shows can be expected if this motel accepts reservations for 5 rooms? for 8 rooms? for 10 rooms? What is the probability that there will not be enough rooms for those who do show up if this motel accepts reservations for 5 rooms? for 8 rooms? for 10 rooms?

42. The Beardstown Ladies wish to invest $1,000 in common stock. They are considering two stocks, Holden stock and Sanborn stock. They both have a normal distribution of returns, as follows: Holden stock ~ N($1100, 1600) Sanborn stock ~ N($1140, 2500). The Ladies only want to select one of the two stocks. Obviously Sanborn stock has a higher expected return, but it is also more risky in the sense that the variance of Sanborn's return is greater than that of Holden. The Beardstown Ladies want to choose the stock that has the smaller probability of returning less than $1000. Which stock should they choose?

43. A normal random variable has a standard deviation $\sigma = 10$. If there is a probability of 0.889 that the random variable will take on a value less than 100, what is the probability that it will take on a value greater than 110? You should first find the mean of X.

44. A random variable has a χ^2 distribution with 8 degrees of freedom. What is the probability that the random variable will be between 13.4 and 17.5? Explain your work.

45. Assume W is a random variable with a *t*-distribution. Find the following probabilities.
a. P[0 ≤ W ≤ 6.31] when W has 1 degree of freedom
b. P[-2.02 ≤ W ≤ 3.36] when W has 5 degree of freedom
c. P[-1.30 ≤ W ≤ 1.30] when W has 40 degrees of freedom
d. P[-1.96 ≤ W ≤ 1.96] when W has ∞ degrees of freedom
Make sure you show your work.

46. Let Ω be distributed F(5,10) and let Ψ be distributed F(10,5). Which of the following probabilities is smaller, P[Ω ≥ 3.3] or P[Ψ ≥ 3.3]? Explain.

47. An investment portfolio contains stocks from a large number of corporations. Over the past year 14% of the corporations had a rate of return over 20%, and 4.5% had a rate of return which was negative. Assuming that these rates of return have a normal distribution, what is the mean, variance, and standard deviation of these rates of return?

48. A professor sees students during regular office hours. The time spent with students follows an exponential distribution with a mean of 10 minutes.
a. Find the probability that a given student spends less than 20 minutes with the professor.
b. Find the probability that a given student spends between 10 and 15 minutes with the professor.

49. Harry and David only accept pears which weigh at least 12 ounces. Their pears have weights which are normally distributed with a mean of 15 ounces. If they reject 12% of their pears, what is the standard deviation and variance of the pear weights?

50. A use car dealer has numerous Toyota Corollas on the lot. The average age of the cars is 2.1 years with a variance of 2.488. If she prices the used cars using the following formula,

Price = 18,947 – 1,788Age

What will be the average price of the cars and the variance of the price?

51. Suppose that X and Y have the following joint probability distribution.

		Y	
		0	1
	-1	0.0	0.25
X	0	0.50	0.0
	1	0.0	0.25

What is the covariance of X and Y? What is the correlation coefficient of X and Y? Are X and Y independent here? Explain.

52. Why does $\frac{(n-1)s^2}{\sigma^2}$ have a χ^2_{n-1} distribution? Do not prove this. Just tell me a story beginning with the definition of a χ^2_n distribution and how the above expression looks very similar.

53. The Cincinnati Enquirer reported that the mean number of hours worked per week by those employed full-time is 43.9. The article further indicated that about one third of those employed full-time work less than 40 hours per week. Assuming the number of hours worked is normally distributed, what is the standard deviation of the number of hours worked?

54. A true-false test consists of 50 questions. How many questions does a student have to get right to convince you that he is not merely guessing. Explain.

55. About 8% of males are colorblind. A researcher needs some colorblind subjects for an experiment, and begins checking potential subjects.
a. on average, how many men should the researcher expect to check to find one who is colorblind?
b. what's the probability that she won't find anyone colorblind among the first 4 men she checks?
c. what's the probability that the first colorblind man found will be the sixth person checked?
d. what's the probability that she finds someone who is colorblind before checking the tenth man?

56. Suppose that the amount of beer in a 12 ounce can has a normal distribution with a mean of 12 ounces and a standard deviation of 1.5 ounces.
a. What is the probability that a randomly selected bottle of beer will contain more than 14.5 ounces of beer?
b. What is the probability that a randomly selected bottle of beer will contain between 9 and 11 ounces of beer?

57. Suppose the weekly hours X of investment bankers is distributed as follows: $X \sim N(90,144)$. The investment club interviews 25 investment bankers and asks them their hours last week. What is the probability that the sample's mean is greater than 91 but less than 93?

58. Suppose that X is a random variable distributed as follows: $X \sim N(150,625)$. A sample of 25 observations is taken. What is the probability that the sample's mean is greater than 140 but less than 147.5?

59. Suppose X is a random variable distributed as follows: $X \sim N(200,400)$. A sample is drawn from an infinite population, and the following probability concerning the sample mean holds:

$$P(\overline{X} \geq 203.92) = 0.25$$

What is the sample size?

60. Suppose $X \sim N(\mu, \sigma^2)$, where $\mu = 0$ and $\sigma^2 = 81$. Calculate the probability $P(\overline{X} \geq 3)$ if $\overline{X}$ is the average of a sample of size n and n is alternatively 9, 64, 81 and 100.

61. Suppose that the number of hours of sleep an American gets is a normally distributed random variable with mean $\mu = 8$ hours. If for a random sample of size 25 Americans

$$P(\overline{X} \geq 8.4) = 0.1587$$

what is the variance of the number of hours of sleep an American gets?

62. The number of hours preschool children spend watching television every week is normally distributed with a standard deviation of 10 hours. A random sample of 4 children was taken in order to estimate the mean number of hours. What is the probability that the sample mean will exceed the population mean by more than 2 hours? What if we increase our sample to 8 children?

63. The number of students who actually come to lecture is normally distributed with a standard deviation of 3.6. A random sample of 4 lectures is taken. What is the probability that the sample variance is bigger than 30?

64. Suppose that the life expectancy of an 18-year-old woman is 80.3 years with a standard deviation of 7.5 years. If a life insurance company writes policies for 45 18-year-old women, what is the probability that their average policy-holder will die before the age of 79.5 years? after the age of 81?

65. Suppose the number of quarts of ice cream consumed annually by an American has a standard deviation of 8 quarts. We select a random sample of 6 Americans in order to estimate the mean number of quarts consumed by all Americans. What is the probability that the sample mean will fall short of the population mean by more than 2 quarts? What if we increase our sample to 9 Americans?

66. Thanksgiving turkey weights are normally distributed with a mean of weight of 15.6 pounds and a standard deviation of 4 pounds. A random sample of 5 turkeys is taken. What is the probability that the sample variance is bigger than 25?

67. Bunge Inc claims that on average there is one defect per 200,000 feet of bunge cord, and the number of defects follows a Poisson distribution. You purchase 100,000 feet of bunge cord for your Bunge Jumping Club.
a. Find the probability that your bunge cord has zero defects.
b. Find the probability that your bunge cord has more than 2 defects.

68. Suppose the average age for a person getting married for the first time is 26 years, and that 15% of people get married before the age of 20. If the ages for first marriage are normally distributed, what is the standard deviation and variance of the ages for first marriage?

69. About 25% of the drivers who are members of the Teamsters Union earn over $18.68 an hour, and about 10% of these drivers earn less than $14.25 an hour. Assuming that these wages have a normal distribution, what is the mean, variance and standard deviation of these wages?