Economics 91 Spring 2013
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**Confidence Intervals and Hypothesis Testing**

I. Confidence Intervals

1. The population of starting salaries for Pitzer graduates is known to have a standard deviation of $3,800. How big a sample is needed to estimate the mean starting salary with 95% confidence and a tolerable error of $100? Of $200?
5548, 1387

2. Campus security is concerned about crime. A random sample of 15 campus thefts shows that the mean amount stolen to $332 with a standard deviation of $136. Construct a 90% confidence interval for the mean stolen amount for all such thefts.
270.16,393.84

3. A pharmaceutical firm is bringing out a new sleeping pill. It is tested on a random sample of 18 people. The sample standard deviation of the time to sleep turns out to be 11 minutes. Construct an 80% confidence interval for the population variance of the time to sleep.
95.23,∞

4. A rental car company is interested in the amount of time its vehicles are out of operation for repair work. A random sample of nine cars showed that over the last year their cars were inoperative for an average of 16.22 days with a standard deviation of 4.79 days. What is the 95% confidence interval? How big a sample is needed to estimate the mean true average number of days out of operation with 95-percent confidence and a tolerable error of 0.1 day?
8815

5. You are asked to spend the next 21 days counting the number of customers who walk into Wolfe's market each day. You find that on average Wolfe's has 230 customers in a day with a variance of 81. Find the 95% confidence interval for the variance.

6. A friend who hears you are taking a statistics course asks for help with a chemistry lab report. She has made four independent measurement of the specific gravity of a compound. The results are:
3.82, 3.93, 3.67, 3.78
The lab manual says that repeated measurements will vary according to a normal distribution with a standard deviation of $\sigma = 0.15$. The mean $\mu$ of the distribution of measurements is the true specific gravity. The lab manual asks for a 95% confidence interval for the true specific gravity. Your friend does not know how to do this. Do it for him. Would the 80% confidence interval be wider or narrower than your 95% confidence interval? (You do not have to compute the 80% confidence interval)

7. A rental car company is interested in the amount of time its vehicles are out of operation for repair work. A random sample of nine cars showed that over the last year their cars were inoperative for an average of 16.22 days with a standard deviation of 4.79 days. What is the 95% confidence interval on the mean time out of operation?

8. According to a Gallup poll, 76% of men and 65% of women claim to be "in love" with someone. The poll surveyed 502 men and 500 women by telephone.
a. construct a 90% confidence interval for the proportion of men "in love."
b. construct a 95% confidence interval for the proportion of women "in love."

9. The annual income of a random sample of five American households is \$15, \$25, \$30, \$40 and \$75 thousand dollars. Assume that household incomes are normally distributed.
a. find the sample mean.
b. find the sample variance.
c. construct a 90% confidence interval for the sample mean.
d. construct a 95% confidence interval for the sample variance.

10. The number of inches of snow in Vermont ski areas is known to be a random variable normally distributed with a known standard deviation of 3 inches. Thirty-six ski areas are randomly and independently selected, and the average number of inches of snow is found to be 102.
a. What is the probability that the 102 inch average of the sample is within 2 inches of the actual population mean? (Assume an infinite number of ski areas).
b. Construct a confidence interval around the mean of the above population (using a 99% confidence level).

11. The weekly income of a random sample of five alumni are \$300, \$400, \$200, \$700 and \$1000. Assuming that the income is normally distributed, find the 99% confidence interval for the mean income of the alumni population.

12. The Mix Bowl wants to estimate the average number of daily customers. The manager wants to have 99% confidence that the error will not exceed 5. It is know that the daily number of customers is normally distributed with a standard deviation of 100. How many days should be sampled?

13. A random sample of 10 coffee drinkers are asked the number of hours they slept the previous night. The responses are 5, 6, 7, 8, 9, 4, 3, 9, 9, and 10 hours.
a. Find a point estimate of the variance of the number of hours of sleep.
b. Assuming that hours of sleep are normally distributed, calculate the 95% confidence interval for the variance.

14. The hourly salary of a random sample of five alumni are \$20, \$30, \$10, \$60 and \$90. Assume that alumni salaries are normally distributed.
a. find the sample mean.
b. find the sample variance.
c. construct a 95% confidence interval for the sample mean.
d. construct a 95% confidence interval for the sample variance.

## II. Hypothesis Testing

1. We want to determine if average motorcycle accident fatality rates are lower in states requiring drivers to wear helmets. In the five states with helmet laws, the average fatality rate was 0.1021 with a standard deviation of 0.0918. In the six states without helmet laws, the average fatality rate was 0.2133 with a standard deviation of 0.0547. Assume that the underlying populations have equal variances. Test the hypothesis that the fatality rates are the same in the two types of states.

2. The standard deviation of incomes is $17,000 in the U.S. and $15,000 in Japan. You sample 10 individuals in the U.S. and find an average income of $23,240. You sample 15 individuals in Japan and find an average income of $28,190. Test the hypothesis that average incomes are the same in Japan and the U.S.

3. Given the sample data below, perform the indicated hypothesis tests at the 5% level.
a. $\overline{X}$ = 13.7, $S_x$ = 2, n = 39, $H_0\!:\mu = 14$.
b. $\overline{X}$ = 93, $S_x$ = 10, n = 42, $H_0\!:\mu = 95$.
c. $\overline{X}$ = 13, $S_x$ = 3, n = 300, $H_0\!:\mu = 18$.
accept, accept, reject

4. For many years, the Federal Aviation Administration has been giving a pilot's exam with a mean score of 85 and a variance of 64. It has introduced a completely new exam and when a sample of 15 persons takes the exam, the sample variance equals 36. Test the hypothesis that the variance of the exam scores for the new exam is the same as for the old at the 5% significance level.
accept

5. A government official wants to determine whether it is true that the average motorcycle-accident fatality rate (average ratio of fatal accidents to all accidents) is lower in states requiring drivers to wear helmets. The record is reviewed. The official finds that 5 states had helmet laws (a) during the entire past decade, while 6 states were without such laws (b) throughout the period. A test at the 5% significance level is to be conducted, using the observed mean fatality rates of $X_a$ = 0.1021 and $X_b$ = 0.2133, along with standard deviations of $S_a$ = 0.0918 and $S_b$ = 0.0547. Make the test, assuming that the underlying populations are normally distributed and have equal variances and that for each state only one figure is available that refers to the entire decade in question.
reject

6. It is claimed that the variance of the weekly income of self-employed lawyers is higher than the variance of the income of lawyers employed by the federal government. Two random samples revealed the following weekly incomes (in $1000):
Self-employed: 1.0, 2.0, 3.0, 6.0 ($\overline{X} = 3, S_x = 2.1602$)
Employed by the federal government: 1.0, 2.0, 3.0 ($\overline{Y} = 2, S_y = 1$)
Assuming normality, use a 1 percent significance level to test the hypothesis $H_0\!:\sigma_x^2 = \sigma_y^2$ against the alternative $H_0\!:\sigma_x^2 > \sigma_y^2$ where $\sigma_x^2$ and $\sigma_y^2$ are, respectively, the variances of lawyers incomes in the two sectors.
accept

7. You are interested in studying the incomes of single women over 80 years of age in a certain large city. In a sample of 20 such women, you find an average income of $17,500 with a standard deviation of $6,200. Test the hypothesis that the true mean is $20,000 at the 5% significance level.
accept

8. Popular culture says that it never rains in Southern California and the temperature here is the same every day. You come to visit Disneyland for three days and record the following temperatures: 75, 80, 70. Test the hypothesis that the true variance in temperatures is no larger than 15.
accept

9. In a sample of 16 American Airlines flights, the planes arrived 10.437 *minutes late* on average with a standard deviation of 8.647 minutes. In a sample of 19 United Airlines flights, the planes arrived 1.474 *minutes early* on average with a standard deviation of 5.680 minutes. Assume that the two variances are the same. Test the hypothesis that American Airlines and United Airlines are equally good at prompt arrivals.
reject

10. The owners of the Great Bear Lake Lodge in Saskatchewan, Canada claim that the lake trout in Canadian lakes is much bigger than the lake trout in American lakes. To prove their contention, the owners weigh a random sample of 31 lake trout caught in Canada one season and find a mean weight of 18 pounds and a sample standard deviation of 12 pounds. The average weight of American lake trout is known to be 15 pounds. Is this sufficient evidence to prove their point? Test the hypothesis that Canadian trout and American trout weigh the same at the 5% level.

11. We take a random sample of 20 female students and find that on average they are 64.5 inches tall with a standard deviation of 2.5 inches. Test the hypothesis that the true variance in heights is no larger than 5 at the 10% level.

12. We want to see if left-handed students and right-handed students have different math SAT scores. In a sample of 19 left-handed students, the average math SAT score was 525 with a standard deviation of 95. In a sample of 23 right-handed students, the average math SAT score was 500 with a standard deviation of 103. Assume that the two variances are the same. Test the hypothesis that left and right handed students perform equally well on the math SAT.

13. The price that women pay for haircuts seems to be much more variable than the price that men pay for haircuts. We sample 8 women and find that on average they spend $21 for a haircut with a standard deviation of $8. We sample 6 men and find that on average they spend $11 for a haircut with a standard deviation of $4.
Assuming normality, use a 1 percent significance level to test the hypothesis $H_0: \sigma_x^2 = \sigma_y^2$ against the alternative $H_A: \sigma_x^2 > \sigma_y^2$ where $\sigma_x^2$ and $\sigma_y^2$ are, respectively, the variance of haircut prices for women and for men.

14. Bags of a certain brand of tortilla chips claim to have a net weight of 14 ounces. Net weights actually vary slightly from bag to bag and are normally distributed with mean $\mu$. A representative of a consumer advocate group wishes to see if there is any evidence that the mean net weight is less than advertised. She selects 16 bags of this brand at random and determines the net weight of each. She finds the sample mean to be $\overline{X} = 13.82$ and the sample standard deviation to be $s = 0.24$. Test the hypothesis that the mean net weight is 14 ounces or more.

15. A researcher wished to compare the average amount of time spent in extracurricular activities by high school students in a suburban school district with that in a school district in a large city. The researcher obtained an simple random sample (SRS) of 60 high school students in a large suburban school district and found the mean time spent in extracurricular activities per week to be $\overline{X} = 6$ hours with a standard deviation of $s_x$=3 hours. The researcher also obtained an independent SRS of 40 high school students in a large city school district and found the mean time spend in extracurricular activities per week to be $\overline{Y} = 4$ hours with a standard deviation $s_y = 2$ hours. Let $\mu_x$ and $\mu_y$ represent the mean amount of time spent in extracurricular activities per week by the populations of all high school students in the suburban and city school districts, respectively. Test the hypothesis $H_0 : \mu_x = \mu_y$ versus the alternative hypothesis that $H_a : \mu_x \neq \mu_y$.

16. A noted psychic was tested for ESP. The psychic was presented with 200 cards face down and asked to determine if each card was one of five symbols: a star, cross, circle, square or three wavy lines. The psychic was correct in 50 cases. Let $p$ represent the probability that the psychic correctly identifies the symbol on the card in a random trial. Suppose you wished to see if there is evidence that the psychic is doing better than just guessing. Test the hypothesis that

$H_0 : p = 0.20$ versus the alternative hypothesis $H_a : p \succ 0.20$.

17. A simple random sample (SRS) of 25 male faculty members at a large university found that 10 felt that the university was supportive of female and minority faculty. An independent SRS of 20 female faculty found that 5 felt that the university was supportive of female and minority faculty. Let $p_1$ and $p_2$ represent the proportion of all male and female faculty, respectively, at the university who felt that the university was supportive of female and minority faculty at the time of the survey. Test the hypothesis that $p_1 = p_2$.?

18. The manufacturer of batteries for aircraft emergency-locator transmitters claims that the lifetime of these batteries is normally distributed with a mean of 30 months. An aircraft manufacturer tests 9 batteries and finds a sample mean of 28.5 months and a sample variance of 4. Test the hypothesis that the manufacturer's claim is true.

19. Maytag claims that their repair personnel are lonely. A random sample of 20 owners of 5-year-old washing machines finds that the mean and the standard deviation of the number of service calls over the 5-year period are 4.3 and 1.8 respectively. Test they hypothesis that the mean number of service calls is actually 6 against the alternative hypothesis that it is less than 6.

20. It is widely believed that autism affects 5% of our children. A recent study of 384 children found that 46 had autism. Is this strong evidence that the level of autism has increased? Test the hypothesis that the rate of autism is indeed 5%.

21. In 1960 the average American man first married at the age of 23.3 years. We randomly select a sample of 40 men, and find that they married at an average age of 24.2 years with a standard deviation of 5.3 years. Test the hypothesis that the average age of first marriage has not changed since 1960.

22. Suppose you record the following temperatures for Los Angeles and Seattle on various days during the course of a year.

Los Angeles: 57, 58, 63, 69, 70, 61
Seattle: 41, 46, 61, 66, 53

a. Test the hypothesis that the average temperature in Los Angeles is 60 degrees at the 5% significance level.
b. Test the hypothesis that the variance in daily temperatures in Los Angeles is 25 versus the alternative hypothesis that the variance is larger than 25 at the 10% significance level.
c. Test the hypothesis that the variance in daily temperatures is the same in Los Angeles and Seattle against the alternative hypothesis that the variance is larger in Seattle at the 5% level.
d. Suppose that we can assume that the variance in temperatures is the same in Los Angeles and Seattle. Test the hypothesis that the mean temperatures in Los Angeles and Seattle are the same at the 10% level.
e. Suppose that we are told that the true variance in temperatures in Los Angeles is 25 and that the true variance in temperatures in Seattle is 81. Test the hypothesis that the mean temperature in Seattle is 5 degrees lower than the mean temperature in Los Angeles at the 5% level.

23. You believe that only half of Americans have confidence in the nation's priests, ministers, and rabbis. The Princeton Religion Research Center surveys 1,206 adults by phone, and finds that 54% give members of the clergy "high" or "very high" marks for honesty and ethics. Should you reject your hypothesis?

24. You have a hypothesis that the average American consumes less than 20 quarts of ice cream every year. You conduct a survey of 24 randomly selected Americans, and ask them how many quarts of ice cream they have consumed in the past year. Your sample mean is 21.9 quarts, and the sample standard deviation is 10.2 quarts. Should you reject your hypothesis?

25. Let the two random variables X and Y represent the number of hours of television watched per week by high school students and college students. Assume they are normally distributed with variances $\sigma_{\mathrm{x}}^2 = 25$ and $\sigma_{\mathrm{y}}^2 = 100$. Two random samples taken from these distributions reveal the following results:

Sample X: 5, 6, 7, 8
Sample Y: 4, 3, 6, 9, 5

Test the hypothesis that the difference between the two means, $\mu_x - \mu_y$, is equal to zero.

26. An examination of the credit card balance on Visa and Discover accounts yields the following figures:

Visa: $n_1 = 26$, $\overline{X}_1 = \$2,000$, $s_1 = \$190$

Discover: $n_2 = 36$, $\overline{X}_2 = \$1,500$, $s_2 = \$200$

where $s_1$ and $s_2$ are the sample estimates of the standard deviation in Visa and Discover accounts respectively. Assume that the two variances are actually the same. Test the hypothesis that the difference between the two means is zero.

27. Let the two random variables X and Y represent the number of hours of television watched per week by high school students and college students. Assume they are normally distributed with variances $\sigma_x^2 = 25$ and $\sigma_y^2 = 100$. Two random samples taken from these distributions reveal the following results:

Sample X: 5, 6, 7, 8

Sample Y: 4, 3, 6, 9, 5

Test the hypothesis that the difference between the two means, $\mu_x - \mu_y$, equals zero.