# AN INVESTIGATION OF PRE-SERVICE SECONDARY MATHEMATICS TEACHERS' DEVELOPMENT AND PARTICIPATION IN ARGUMENTATION

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*Abstract: This study investigates how two professors and pre-service secondary mathematics teachers engage in argumentation and proof in two courses. One course under investigation is a geometry course; the second is a methods of teaching mathematics course. The research also studies the how professors and pre-service teachers construct arguments and proofs. Examining the classroom discourse to understand how it may impact argumentation practices is another aspect of the research. Case study and grounded theory approaches are used to guide the data collection and analysis. Some data collected include interviews with the two professors and pre-service teachers and observations of the two courses and the pre-service teachers' classrooms during their student teaching. Data analysis so far indicates the geometry professor engages students in argumentation and proof in multiple ways.*

*Key words:* Pre-service teachers, Mathematics, Proof, Argumentation

## Introduction and Literature Review

According to Forman, Larreamendy-Joerns, Stein, and Brown, "students must also learn about the nature of mathematical argumentation as they attempt to use it to gain an understanding of the mathematical objects under discussion" (1998, p. 529-530). In other words, an aspect of learning mathematics is learning the domain's argumentation practices. Walshaw and Anthony claim argumentation practices in mathematical discourse are "a defining feature of quality classroom experience" (2008, p. 516). Thus, the teaching of proof is valuable for learning mathematics, but we have little research about it (Stylianides, 2007). Bass questions if proof is an "endangered species," which is a form of argumentation in mathematics (2011, p. 98). Uhlig (2002) claims students have little experiences with proof before college so it is plausible that teachers place little emphasis on proof when teaching. Cross (2009) believes engaging in argumentation can foster and help disseminate mathematical ideas; in particular, practices such as conjecturing are those classroom discourse can support. Yet Cross (2009) claims teachers need to model and help students understand the argumentation practices they want students to adopt.

The Common Core State Standards (2010) acknowledge proof and argumentation as a valuable part of students' mathematical learning. These standards outline opportunities students should have related to argumentation in their standard "Construct viable arguments and critique the reasoning of others"; this includes giving students opportunities to conjecture. Some elements of the standard of mathematical practices, besides conjecturing, are to provide time for students to "Constructing arguments", "recognize and use counterexamples, and "make plausible arguments". Evidence of the value they place on argumentation is that in every grade listed practices related to argumentation are cited. Teachers need the ability to formulate strong mathematical arguments and proofs because they must respond to students' mathematical claims or explanations. Walshaw and Anthony claim, "effective pedagogy is inclusive and

demands careful attention to students' articulation of ideas" (2008, p. 527). Thus, the experiences teachers' design for mathematical argumentation has pedagogical importance. In fact, Krummheuer (2007) considers mathematical argumentation an everyday activity in the mathematics classroom. Teachers are expected to teach proof and argumentation and engage students in mathematical argumentation. Thus, teaching and fostering argumentation and proof supports recently adopted standards (see Common Core State Standards, 2010).

The focus of the research is to study the mathematical argumentation of two professors and pre-service secondary mathematics teachers enrolled in two courses- a methods of teaching mathematics course and a geometry course. The research pursued here is guided by the following research questions: How do the students and teacher engage in argumentation in mathematics? How are arguments constructed? How do argumentation practices develop over time? How does the classroom discourse around mathematics influence argumentation? Some "students" in the geometry course will be identified as "teachers" once they begin their student teaching.

## Methodology

Data collection began in the fall semester of 2012 and will continue through May 2012 and so I will continue to collect data after submission of the proposal. Thus, the analysis presented here is based on data collected up to this time. Observations of the courses and suggestions from the professor will be taken into account for selecting students to participate in interviews and classroom observations. Open and in-vivo coding will be conducted for initial coding stages of the interview data of the professor and students, which are at present fully transcribed. Triangulation will be used in data analysis, which according to Merriam "remains a principle strategy to ensure for validity and reliability" (2009, p. 216). Also, member checks, and efforts to establish researcher reflexivity (e.g., Cho & Trent, 2006; Merriam, 2009) will be used.

The research study's participants include two professors in education at a university in the Rocky Mountain region and undergraduate pre-service secondary mathematics teachers. One professor is the instructor of a geometry course which the pre-service teachers (henceforth called students) are enrolled for Fall 2012. This course was chosen for several reasons, but one is that it is an advanced level mathematics course and so forms of argumentation such as proof and counterexamples are likely to be encountered. Many of the students are concurrently enrolled in the methods and geometry course. Thus, the possibility to observe how teaching proof and argumentation is addressed was a reason for choosing the methods course.

Data consist of field notes from observing the two courses, interviews with the professors and selected students, written work collected from students, and observations of classroom visits to see the students teach in their own classrooms. Interviews throughout the semester with the professors are based on observations of the course and written work produced by students. Questions posed during interviews with professor are meant to draw out information regarding how they plan to and did engage students in argumentation and proof, forms of argumentation they saw students using, and how students constructed proofs and mathematical arguments. Students selected for interviews and observations have not been chosen yet, but will be chosen based on the forms of argumentation they may have employed, questioned, or in the way in which they responded to a given mathematical argument. Interviews with students will focus on

forms of argumentation and proof, how they engaged in proof and argumentation, and explore how they constructed them.

Grounded theory and case study approaches are employed. Because a small number of students will be selected (possibly two or three) to participate in interviews and observations of their teaching practice, the phenomenon under investigation is considered "intrinsically bounded", a criteria for case study (Merriam, 2009, p. 41). Stylianou, Blanton, and Knuth (2011) claim there is little research on how proof is taught in schools. Thus, observing pre-service teachers during their student-teaching experiences can help address this gap in the research. Each professor and student will be considered as separate cases. The interactions between professors and students will provide valuable data concerning the engagement, teaching, use, and development of mathematical arguments and proofs. This context represents one described by Grbich as "interactions between persons or among individuals and specific environments", which justifies grounded theory is a suitable approach (2007, p. 70). Also, because little is known how teachers develop arguments in mathematics, it is another reason why grounded theory is an appropriate approach (Grbich, 2007). Toulmin's model of argumentation has been used by numerous researchers (e.g., Giannakoulias, Mastorides, Potari, & Zachariades, 2010; Krummheuer, 2007; Pedemonte & Reid, 2011), to analyze the structure or proofs and arguments in mathematics and will be used for this research.

## Preliminary Findings and Discussion

My current data set consists of field notes of course observations, documents from the course (e.g., syllabi, assignments), and interview data (transcripts). Preliminary analysis of observations indicates there is a variety of ways the geometry professor engages students in argumentation practices. One approach he has used often is to make historical references to proof. These reference highlight mathematical claims individuals have attempted to prove throughout time and the changing emphasis of proof in schools. Also, he related the structure of mathematics to proof. An instance of this is one of the discussions of non-Euclidean geometries such as Lobachevski, based on a restatement of Euclid's fifth postulate. An unexpected change in the acceptable form of proof took place after the second quiz. The instructor noticed many students struggled with constructing proofs on the quiz; all students wrote proofs in a narrative form, according to the professor, but many with circular reasoning or an absence of justifications for assertions. Thus, from that point on the professor required students to write proofs on quizzes and exams in the two-column format until he felt confident they could construct narrative proofs that provided assertions followed by justifications. His description of expectations for two-column proofs is consistent with Weber and Alcock's notion of proof, which "must be based on accepted axioms and definitions" (2011, p. 323). To analyze students' arguments and proofs, Toulmin's model may be used.

Questions for the audience:

1. What might be important concepts that lend themselves well to studying proof and argumentation?
2. The use of Toulmin's model of argumentation has been used by many other researchers. Are there other models or frameworks better fitting to this research?

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