A MULTIDIMENSIONAL ANALYSIS OF INSTRUCTOR QUESTIONS IN ADVANCED MATHEMATICS LECTURES

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This study is an investigation of the questions that are asked by four faculty members who were teaching advanced mathematics. Each question was analyzed along three dimensions: the expected response type of the question, the Bloom's Taxonomy level, and the context of the question within the mathematics content.

Key words: Teaching practices, questions, cognitive engagement, Bloom's Taxonomy

Introduction

Instructional practices at the undergraduate level have been largely unexamined (Speer, Smith, & Horvath, 2010) and many of the studies that do focus on teaching practice have occurred in lower division courses like calculus, where students are expected to be able to do computations and applications (Thompson, et.al. 2007, Epstein, 2007; Bressoud, 2011). In advanced mathematics courses, the content shifts to formal mathematics, and undergraduates are expected to be able to comprehend and write mathematical proofs. Because the content and expectation of the students is quite different, the teaching of mathematics at this level may need to be examined separately. Previous studies of teaching practice at this level have focused on proof presentation in class (Fukawa-Connelly 2012a; Fukawa-Connelly 2012b; Mills, 2012; Weber, 2004). This study will add to the existing literature by examining the interactions between the instructor and students in advanced undergraduate mathematics courses.

The participants of this study are four instructors teaching different upper-division proof courses at the undergraduate level. All of these instructors taught using some variation of lecture, meaning that the instructor was primarily standing at the board presenting the material while the students were sitting in desks. Studies that examine teaching often focus on where the instructor's presentation method lies on the continuum of lecture to reform (Sawada, Piburn, Judson, Turley, Falconer, Binford & Bloom, 2002; Steussey, 2006; McClain & Cobb, 2001), but this emphasis on the presentation style tends to gloss over subtle features of teaching practice, namely, differences that may occur within a lecture format. This study will provide a multi-dimensional analysis of the questions that are used by these instructors while lecturing.

Research Questions

- How often do instructors who are teaching advanced mathematics using lecture methods interact with their students by asking questions?
- What types of questions are asked by instructors who are teaching advanced mathematics using lecture methods, and what types of responses are expected of students?
- Does the mathematical content presented (definitions, theorems, proofs, examples) have an effect on the frequency or type of questions that are asked?

Literature Review

Lecture is still widely used in undergraduate mathematics instruction, and a majority of instructors believe that lectures can be effective (Bressoud, 2011). However, few studies investigate in detail the teaching practices of instructors using primarily lecture methods. As Krantz (1999) points out, a masterful lecturer may include many different pedagogical moves to connect to his or her audience. Instructors can use examples, give summaries, check for student understanding, or make connections between different topics (McKeachie & Svinicki, 2006). Lecture can also be interactive, incorporating lots of questions that guide students through the material (Bagnato, 1973). In short, there can be significant variation among lecturers.

This study will focus on the teacher-student interactions in advanced mathematics lectures. The literature on interactions in mathematics classes seems to fall into two camps: those that look for overarching patterns in interactions (Fraivillig, 1999; Fukawa-Connelly, 2012a; Henningsen & Stein, 1997; Lobato et al, 2005; Tobin, 1986;), and those that classify the types of questions used by instructors and students (Gall, 1970; Sahin & Kulm, 2008; VanZee & Minstrell, 1997; Wood, 1994; Wood, 1999).

Several existing taxonomies, such as Bloom's Taxonomy, record the cognitive level that the student will need to use to answer the question (Anderson & Krathwohl, 2001; Gall, 1970; Tallman & Carlson, 2012). Gall (1970) points out that there are several types of questions do not fit well into these taxonomies. In particular, in this study, rhetorical questions and general questions that check for student understanding often do not fit well into these taxonomies. Other classification schemes classify questions by the expected products, such as whether the questions require the student to make a choice, give factual information, give reasons for their thinking, or justify their thinking (Wood, 1999; Mehan, 1979). Although this is a reasonable way to catalog question types, the expected response type does not necessarily capture the cognitive processes required to answer the question. This study will consider the expected response types and cognitive engagement as separate dimensions, which will allow for a more in-depth classification of the question types.

Although there are many studies investigating teacher-student interactions and questioning patterns, there are few that investigate the teaching of advanced mathematics. One study investigates how one instructor of advanced mathematics, Dr. Tripp, used questions to devolve responsibility to students when presenting proofs (Fukawa-Connelly, 2012a). This case study showed that Dr. Tripp often used linked questions that both modeled her proof writing strategy and reduced the cognitive demand for the students. She often began with a higher-level question but then asked several successive questions in a row until the final question required merely a factual response or re-statement of something she previously said. Though she did devolve some responsibility for proof writing to students, the majority of students' answers stated the next part of the proof or the next algebraic step.

In his discussion of Dr. Tripp's questioning patterns, Fukawa-Connelly (2012a) describes some of her questions as "high level" or "factual," but this study contributes by providing more specific and detailed descriptions of the questions that were asked in lectures. Bloom's taxonomy has been used to investigate the types of questions that appear on undergraduate Calculus exams (Tallman & Carlson, 2012). This study differs because the abstract nature of the mathematics in these courses is quite different from the content in Calculus courses, and therefore the types of questions asked may be different. This study also investigates instructors' questions that are used in the context of teaching, not questions that have been constructed specifically for use on examinations.

Methods

Video observation data were collected periodically during the regular semester in each of four courses: Geometry, Number Theory, Introduction to Modern Algebra, and Introduction to Modern Analysis. Six observations of each classroom were conducted, with the camera focusing on the instructor and the chalkboard. The observation days were chosen so that they occurred on instruction days, and were spread out so that they occurred approximately every two weeks throughout the semester. More information about how the data were collected can be found in Mills (2012).

Any instance in the video data where the instructor either requested information from a student or posed a rhetorical question to the class was transcribed. For each question, the expected response type was recorded. The question was coded as rhetorical if the instructor either answered it himself immediately after posing it, or if he did not wait for the students to respond to the question. Questions such as "Does that make sense?" and "Any questions?" were coded as comprehension questions. The other codes for expected response types (choice, product, process, meta-process) came from Mehan (1979), and are described in the Appendix. Each question was also analyzed to determine its cognitive level based on Bloom's Taxonomy (Anderson & Krathwohl, 2001), however, their "remember" category was parsed into "remember" and "apply a procedure" as in Tallman & Carlson's framework (2012). For each question, I also noted the mathematics context (definition, theorem, example, or proof) which describes what the instructor was presenting at the time that the question was posed.

Preliminary Results

To this point, two videos from each instructor have been coded. A two-dimensional analysis of the data is presented in Table 1. Table 1 combines all of the questions that were asked by the four instructors in the coded observations, and situates them in the table based on the expected response type and cognitive engagement. Previous research in undergraduate mathematics courses has stated that the majority of questions asked are lower-level questions, but this data shows that higher level questions appear frequently in these lectures of advanced mathematics.

Table 1: Expected response type vs. Bloom's level for Examples

	N/A	RE	AP	U	AU	A	EV	CR
RQ	24	12	8	16	7	8	1	0
CQ	65	1	0	0	0	0	0	0
CHQ	0	8	2	5	0	3	0	0
PDQ	0	20	16	21	34	9	1	0
PRQ	0	0	6	25	32	46	14	11
MPQ	0	0	0	7	1	6	3	0

The data also show a high number of rhetorical questions and comprehension questions. The preliminary analysis seems to show a difference between instructors with respect to the number of rhetorical questions asked. This analysis also does not take into account linked questions, and so it could be that higher-level questions are immediately followed by lower-level questions, as observed by Fukawa-Connelly (2012a). To investigate this, another dimension that records whether or not a student responds to the question may be beneficial in the analysis as well. As the remainder of the observations are coded, more patterns in the cognitive engagement, expected response types, and mathematics context will be reported.

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Appendix

Expected Response Type

CQ Comprehension question: The instructor checks for understanding (e.g., "Does that make sense?") and pauses for at least two seconds, thereby indicating an opportunity for students to respond.
RQ Rhetorical question: The instructor asks a question without seeking an answer and without giving students an opportunity to answer the question.
CHQ Choice question: Instructor asks a question that dictates that the student agree or disagree with a given statement.
PDQ Product question: Instructor asks a question which requires students to provide factual responses.
PRQ Process question: Instructor asks a question which calls for the students' opinions or interpretations.
MPQ Meta-process question: Instructor asks a question which requires students to reflect on their thinking and to formulate the grounds for their reasoning.

Cognitive Engagement (Bloom's Taxonomy Categories)

RE Remember: Students are prompted to retrieve knowledge from long term memory.
AP Apply a procedure: Students must recognize and apply a procedure.
U Understand: Students are prompted to make interpretations, provide explanations, make comparisons, or make inferences that require understanding of a mathematics concept.
AU Apply understanding: Students must recognize when to use a concept when responding to a question or when working a problem.
A Analyze: Students are prompted to break material into constituent parts and determine how the parts relate to one another and to an overall structure or purpose.
EV Evaluate: Students are prompted to make judgments based on criteria and standards. Checking and critiquing are characteristic processes at this level.
CR Create: Students are prompted to reorganize elements into a new pattern or structure. Generating, planning, and producing are characteristic processes at this level.

Context

DEF Definition: The question was asked during the presentation of a definition.
THM Theorem: The question was asked during the presentation of a theorem.
EX Example: The question was asked during the presentation of an example.
PF Proof: The question was asked during the presentation of a proof.