PRESERVICE ELEMENTARY TEACHERS' UNDERSTANDING OF GREATEST COMMON FACTOR STORY PROBLEMS

Kristin Noblet
University of Northern Colorado

Little is known about preservice elementary teachers' mathematical knowledge for teaching number theory concepts, especially greatest common factor. As part of a larger case study investigating preservice elementary teachers' understanding of topics in number theory, both content knowledge and pedagogical content knowledge (Shulman, 1986), a theoretical model for how preservice elementary teachers understand GCF story problems was developed. An emergent perspective (Cobb & Yackel, 1996) was used to collect and analyze data in the form of field notes, student coursework, and responses to task-based one-on-one interviews. The model resulted from six participants' responses to three sets of interview tasks where participants discussed concrete, visual, and story problem representations of GCF. In addition to discussing the model and relevant empirical evidence, I suggest language with which to discuss GCF representations.

Key words: Preservice Elementary Teachers, Story Problems, Number Theory

Many preservice elementary teachers have a limited understanding of the mathematics that they will teach, including many topics in number theory (e.g., Zazkis & Liljedahl, 2004), which suggests that they may not be prepared to teach mathematics for understanding. The research also suggests that pedagogical content knowledge (PCK) and specialized content knowledge (SCK) are important for teaching (e.g., Ball, Thames, & Phelps, 2008; Shulman, 1986), but little is known about preservice elementary teachers' PCK and SCK in number theory. While the literature partially addresses preservice teachers' understandings of number theory content such as evens and odds (Zazkis, 1998), multiplicative structure (Zazkis & Campbell, 1996), primes (Zazkis & Liljedahl, 2004), and least common multiple (Brown, Thomas, & Tolias, 2002), preservice elementary teachers' understanding of greatest common factor (GCF) has yet to be investigated.

My overarching research question was: What is the nature of preservice elementary teachers' understanding of topics in number theory? 'Understanding' pertains to both understanding content (primarily SCK) and PCK (Shulman, 1986; Ball, Thames, & Phelps, 2008) of number theory topics such as GCF, least common multiple (LCM), prime numbers, prime factorization, and congruences. As little is known about preservice elementary teachers' understanding of GCF in particular, this report will focus on this topic.

Methodology

I conducted an interpretive case study (Merriam, 1998) of preservice elementary teachers with a mathematics concentration enrolled in a number theory course. Data for this study came from multiple sources: classroom observational notes, student coursework, as well as responses from two sets of one-on-one, task-based interviews. This report focuses on the development of a theoretical model that emerged from the six interview participants' (Brit, Cara, Eden, Gwen, Isla, and Lucy) responses to three sets of tasks pertaining to GCF representations.

The first set of tasks, posed during the first interview, asked participants to (1) create a GCF story problem that would require one to determine the GCF of 28 and 32, (2) create or describe representations for the GCF of 28 and 32 using manipulatives or pictures, and (3) identify GCF story problems from a list of given story problems. Participants' responses to this set of tasks led me to pose two follow-up sets of tasks during the second interview. First, I asked participants to create concrete, visual, and story problem representations for division and to identify valid division story problems. Finally, I asked participants to validate story problems created by hypothetical students.

An emergent perspective (Cobb & Yackel, 1996) served as my lens for collecting and analyzing data. I primarily used the psychological lens to analyze the data described in this report, since my model is based on individual conceptions about GCF representations. Constant-comparative coding (Corbin & Strauss, 2008) was used as part of the coding process. After determining emergent themes, I found that quite a few of these codes contributed to participants' understanding of GCF story problems, a model for which I describe here.

Results

Participants did not have an opportunity to create, or even answer, GCF story problems in their number theory class. They were, however, given the opportunity to briefly explore LCM visual, concrete, and story problem representations while working on an assignment. Some participants designed tasks that required students to use Cuisenaire rods to find the LCM of two numbers while others wrote story problems involving LCM in some way. Their limited experience with GCF story problems presented me with a unique opportunity during the interviews to observe their processes for trying to understand this novel concept. In the next few sections, I generalize this process using specific examples from the interviews.

Creating Concrete and Visual Representations of GCF

The first GCF task I posed to participants was to create a GCF story problem. Aside from Brit, who immediately attempted to create a story problem, participants engaged in different activities to help them respond to this task. Eden and Isla verbally recalled the basic definition of GCF; others used numerical methods to find the GCF of 28 and 32. For instance, Cara and Eden found the GCF by selecting the largest common factor after listing all of the factors of 28 and 32. Isla and Lucy discussed how to use factor trees to find the GCF of two numbers. Participants switched strategies quickly, as these did not appear sufficient for creating story problems. Most spontaneously created visual or concrete representations of GCF with which to inspire their story problems.For instance, after finding the GCF of 28 and 32, Cara described how to break up 28 and 32 objects to show the GCF.

Cara: So you would end up with 4 groups of a certain number in it. So for 28, you would have 4 groups of 7, and with 32 you would have 4 groups of 8. So the number in your groups would be different, but the amount of groups is the same, showing that that represents the [greatest] common divisor.

Gwen had a different approach for representing GCF, and she even began discussing how she might use this representation to create a story problem.

Gwen: So maybe we have 28 objects and 32 objects… make them into equal groups with the same amount in each group for… So there's going to be 8… this is going to have 7… I'm thinking that this will show that there's 4 in each one. But I would have to word it in a way that would make sense that there are equal groups in each one for the numbers 28 and 32 to have the same amount in each group...

While Cara and Gwen both accurately represented the GCF of 28 and 32, neither one described how you might *find* the GCF. Instead, they first found the GCF, then broke up the groups of 28 and 32 objects into smaller groups using the GCF. This process de-emphasizes the importance of maximizing the common factor, and it proved to be problematic when they attempted to create story problems from their representations. Brit, Isla, and Lucy described how they might use their representations to find the GCF, and they all attempted to account for maximizing the common factor in their story problems.

As exemplified by Cara's and Gwen's responses, participants created two types of GCF representations, each one drawing from a different meaning of division. While the literature frequently refers to these two meanings or models of division as *partititve* and *measurement* (e.g., Ball, 1990), participants were familiar with Beckmann (2008), so the language I suggest here draws from her terminology. Beckmann refers to these meanings as the "How many groups?" meaning of division, where the quotient is represented by the number of groups you can make and the divisor determines the size of the groups, and the "How many in each group?" meaning of division, when the quotient is represented by the number of objects in each group and the divisor determines the number of groups. While intuitively it makes sense that there should also be two meanings of GCF, a review of relevant textbook materials and research did not reveal language with which to discuss them.

We determine the GCF of two numbers, *A* and *B*, by breaking down *A* objects and *B* objects into equal groups (either an equal number of groups or equal sized groups). However, participants frequently referred to the *A* objects and the *B* objects as "groups" as well. To avoid confusion, I reserve the term "group" for the groups of *A* or *B* objects. I refer to the smaller groups that amount to *A* or *B* objects as "subgroups". Thus, for the purposes of this study and drawing from Beckmann's (2008) phrasing, I refer to the two meanings of GCF as "How many subgroups?" and "How many in each subgroup?". Besides Cara, Lucy was the only other participant to create a "How many subgroups?" representation of GCF. Gwen, Brit, Cara, Eden, and Isla created "How many in each subgroup?" representations of GCF.

I suspected participants' understanding of division to be connected to their understanding of GCF due to the connection between the meanings of division and GCF. For instance, I suspected that Lucy, who demonstrated a strong understanding of and a strong inclination towards the "How many subgroups?" meaning of GCF, would be more inclined towards the "How many groups?" meaning of division. To investigate this, I posed tasks pertaining to various representations of division during the second interview. Surprisingly, Isla and Lucy demonstrated an inclination towards the other meaning of division.

Creating GCF Story Problems

Aside from Eden, all of the interview participants attempted to create GCF story problems, with varying degrees of success. A GCF story problem should maintain a GCF structure (groups of objects broken into subgroups, where the number of subgroups or the number of objects in each subgroup is maximized), but there are other things to consider; the narrative of a story problem contextualizes the structure of a mathematical concept, and a story problem poses a question related to this concept for students to answer. For GCF story problems, this question should be precise enough that the only answer is the GCF. Additionally, to ensure that the story problem is as authentic to real life as possible, the context should necessitate the conditions of the mathematical structure somehow. With GCF story problems, it is not enough to describe breaking up groups of objects into smaller groups; the context of more authentic story problems presents a reason for doing so.

Unsurprisingly, participants' GCF story problems drew from the meaning of GCF they used to create their visual or concrete representations of GCF. Lucy created a "How many subgroups?" GCF story problem, while Brit, Gwen, and Isla created "How many in each subgroup?" GCF story problems. Cara was the only participant to create both types of story problems. It is important for the factoring structure of the story problem to specify that all objects are used by the subgroups, but Brit and Isla were the only participants to explicitly mention this. All of the interview participants sufficiently established that they were looking for common factors by stating that the number of subgroups or the size of the subgroups between groups should be the same. However, Cara and Gwen neglected to include a statement maximizing the common factor. Recall that both Cara and Gwen deemphasized maximizing the common factor in their visual or concrete representations of GCF. Brit, Isla, and Lucy used the words "greatest", "largest number", and "highest number", respectively, in their story problems to indicate that they were looking for the GCF. However their phrasing was choppy or unclear, which may indicate difficulty in contextualizing this condition.

While Brit's story problem was somewhat unclear and required clarification, it was perhaps the most contextualized of the story problems.

Brit: I have 28 dinosaur stickers and 32 flower stickers and I want to group the dinosaur stickers and the flower stickers together… and I want to give them to individual students. So I want to know what is the greatest… how many, how many dinosaur stickers and flower stickers am I going to need in each group? I want to use all of them in an equal amount of groups. So I want to know how many stickers are going to be in each group.

Not only were the numbers 28 and 32 put into a context, but most of her conditions were also phrased consistently with this context. Maximizing the common factors was the only condition that she neglected to phrase in context. Cara's story problems were similarly contextualized, but it lacks reasoning for grouping objects the way Cara suggested. Gwen, Isla, and Lucy, however, posed story problems that were contrived and barely contextualized. As an example, consider Lucy's story problem below.

Lucy: Someone has 32 marbles and then student B has 28 marbles. Can they divide them into the same amount of groups, like the highest number, the same amount of groups?

The only discernable difference between Gwen, Isla, and Lucy visual or concrete representations and their story problems was that they posed a question for students to answer. Lucy's question, however, was vague and would not result in the GCF. Isla and Cara posed similarly vague questions. While the other participants posed more specific questions, they would result in common factors rather than the GCF, specifically.

Validating GCF Story Problems

Shortly after I asked participants to create GCF story problems, I asked them to identify valid GCF story problems from a list of four story problems, three of which were valid. The first of these three story problems was structured using the "How many subgroups?" meaning of GCF, while the other two used the "How many in each subgroup?" meaning of GCF. Cara and Lucy, the participants that created "How many subgroups?" representations of GCF, were the quickest to recognize the validity of the first story problem. Gwen eventually solved the story problem to validate it, Isla thought it might be valid but could not explain why, and Eden was not sure. Brit, however, was convinced that it was not a valid story problem because it was "asking the wrong question", i.e., a "How many subgroups?" question. Similarly, Cara and Lucy determined that the "How many in each subgroup?" story problems were asking the wrong questions. This was surprising considering Cara's success in modeling GCF representations using both meanings. The other participants were more likely to correctly identify the "How many in each subgroup?" story problems, as they created similar representations themselves.

While this task helped me to identify participants' predilection to a certain meaning of GCF, it did not provide participants with a sufficient opportunity for discussing the various minutiae involved with GCF story problem design. Thus, in the second interview, I asked participants to critique hypothetical student story problems with various issues, inspired by the participants themselves. One story problem did not maximize the common factor, while the other maximized the wrong factor, not all of the objects were used, and posed an incorrect question. Both of these story problems drew from the "How many in each subgroup?" meaning of GCF, as more participants demonstrated success in this type of representation. Surprisingly, in spite of this success, participants incorrectly determined that these story problems were, for the most part, valid. Brit was the only participant to accurately identify more than one issue. Lucy incorrectly determined the second story problem to be valid since it posed a "How many subgroups?" question, indicative of the "How many subgroups?" structure she was oriented towards. In general, the results of the validation tasks implied that success in creating GCF story problems was not indicative of success in validating them, or vice versa, since Eden felt unable to create a GCF story problem but demonstrated some success in validating them.

Theoretical Model for Understanding GCF Story Problems

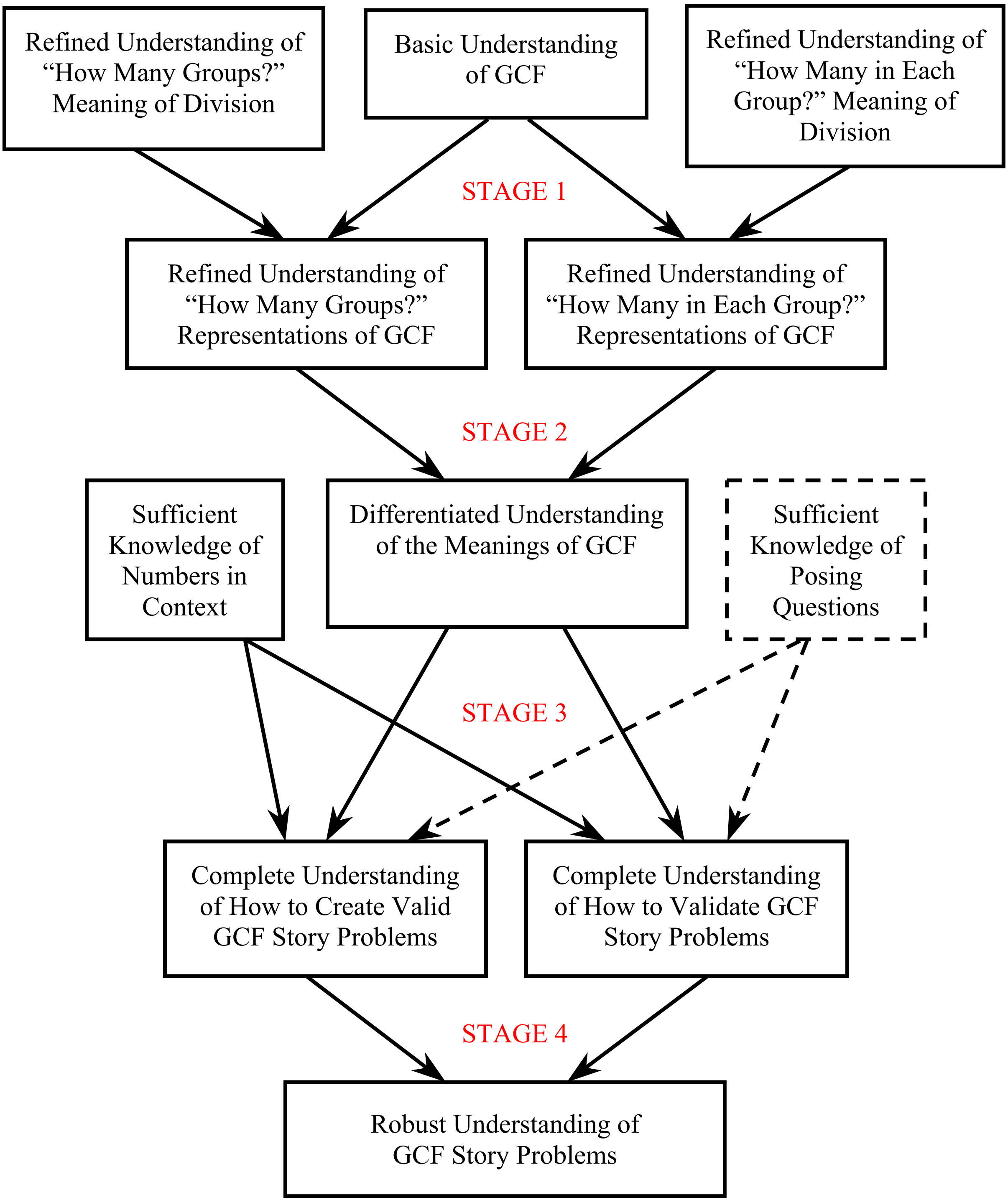

Figure 1. Preservice elementary teachers' process for developing a robust understanding of GCF story problems

The figure above is a proposed model for preservice elementary teachers' process for developing a robust understanding of GCF story problems. Due to the similarities in their representations, it is clear that representing GCF is connected to and understanding of the meanings of division. However, as evidenced by Isla and Lucy specifically, this connection is not always made, which can hinder preservice elementary teachers' understanding of GCF. This suggests the importance of making clear connections between the meanings of division and the basic definition of GCF, Stage 1 in the model, to help students develop a clear understanding of the different representations of GCF and their structures. Participants who demonstrated finding the GCF using their representation had a better understanding of the representation than the participants who used the GCF value to visually divide the original two numbers. This distinction proved to be important when participants created story problems, so using their representations to find GCF should be emphasized in scaffolding preservice elementary teachers' understanding of GCF representations.

Most participants did not represent GCF using the two different meanings, and thus did not reconcile the differences and similarities between the two before attempting to create or validate a GCF story problem. As a result, these participants partially conflated the two meanings of GCF in their story problems due to ambiguous wording, they struggled to validate story problems dissimilar from their own, and they struggled to identify students' mistakes in creating GCF story problems. This suggests that comparing and contrasting the two types of GCF representations, Stage 2, thus encouraging the development of a differentiate understanding of GCF, may facilitate keeping track of the various minutiae involved with GCF story problems.

Even though Cara demonstrated a relatively differentiated understanding of GCF, having successfully created and compared both types of representations, she had limited success in validating GCF story problems because she struggled understanding the GCF structure in context. Furthermore, Eden, Gwen, Isla, and Lucy all struggled, to varying degrees, contextualizing their GCF representations. This suggests that a differentiated understanding alone is insufficient, and that perhaps participants require some understanding of numbers in context to create and validate GCF story problems. Half of the participants also posed vague questions in their story problems or incorrectly validated the questions posed in given story problems. It is unclear if this is due to a weak understanding of the meanings of GCF, or if preservice elementary teachers' understanding of GCF story problems might benefit from a general understanding of how to pose questions. Regardless, in Stage 3 of the model, it is important that preservice elementary teachers negotiate their understanding of GCF with their understanding of how numbers behave in context to gain more complete understandings of how to create and validate GCF story problems.

While participants simultaneously developed understandings pertaining to validating and creating GCF story problems in Stage 3, I propose that it is not until they have successfully done both of these things and reconciled the two types of experiences, Stage 4, that they will have a robust understanding of GCF story problems. It is even possible that an interplay between the two concept images is necessary before either one is robust.

Discussion and Implications

The research pertaining to how preservice elementary teachers understand story problems and how to create them is limited. However, the researchers who have investigated this phenomenon in part (e.g., Ball, 1990; Crespo, 2003; Goodson-Espy, 2009) have found that preservice elementary teachers tend to struggle to represent mathematics contextually through story problems. Understanding story problems, creating and critiquing them, lies within the realm of specialized content knowledge or SCK (Ball, Thames, & Phelps, 2008). Creating and critiquing story problems requires content knowledge specific to teachers, but as many of my participants acknowledged, story problems are also useful in "helping kids understand" concepts, suggesting that their use in the classroom may demonstrate pedagogical content knowledge or PCK (Shulman, 1986). As story problems can be an important pedagogical tool, especially in elementary school, the proposed model has implications in elementary teacher education. While this model specifically informs teacher educators on how to best scaffold preservice elementary teachers' understanding of GCF story problems, it is likely that a similar process might be useful for understanding number and operations story problems in general.

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