REASONING ABILITIES THAT SUPPORT STUDENTS IN DEVELOPING MEANINGFUL FORMULAS TO RELATE QUANTITIES IN AN APPLIED PROBLEM CONTEXT

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This poster illustrates and describes student thinking when responding to applied problems to relate two quantities that cannot be directly related by a single formula. Students who understood the meaning of the directive to define one quantity in terms of another, and who also conceptualized variables as representing varying values that a quantity can assume were successful in constructing a meaningful formula to relate the values of two quantities that cannot be directly related by a single formula. Students who failed to construct meaningful formulas during their solution process either held the view that a variable is an unknown value to be solved for or could not meaningfully interpret the directive of the problem statement.

Key words: Function, Quantity, Variable

Introduction And Theoretical Framework

It has been widely reported that a process view of function is critical for understanding and using ideas of function composition and function inverse (Breidenbach, Dubinsky, Hawks, & Nichols, 1992). Students who do not conceptualize a function as relating a continuum of values in a function's domain to a continuum of values in a function's range are unable to imagine linking two function processes together for the purpose of relating two quantities that cannot be related by a single formula. The purpose of this study was to reveal how students' conceptions of quantity, variable, and function impact their reasoning when responding to a problem that required the use of function composition to relate two quantities in an applied problem.

Methods

We administered two applied problems (Figure 1) to 123 precalculus level students and used open coding (Strauss & Corbin, 1990) to analyze student written responses. We then conducted clinical interviews with 5 of these students to characterize their thinking relative to their conceptions of quantity, variable, and function and how these conceptions impacted their solution approach. The clinical interviews followed the methodology described by Goldin's (2000) principles of structured, task-based interviews, including metacognitive questions to reveal the reasoning processes students used to determine their answers.

Task 1:	The perimeter of a rectangle is 40 feet and the length of one side of the rectangle is 8 feet. Determine the area of the rectangle.
Task 2:	The perimeter of a rectangle is 40 feet and the length of one side of the rectangle is w feet. Express the area of the rectangle A in terms of the rectangle's width w.

Figure 1. Assessment and clinical interview tasks.

Results and Findings

Of the 123 students, 107 provided a correct answer to the first task, and of these 107 students, only 46 provided a correct answer to the second task. Of the 61 students who did not answer the second question correctly, only 4 advanced their solution to the point of expressing the length of the rectangle in terms of the width (e.g., $l = 20 - w$), suggesting that this construction is key to students' advancing their solution towards expressing the area of the rectangle in terms of the width of the rectangle. Analysis of interview data revealed that in order for students to necessitate relating length and width in a single formula, they needed to have a process view of function. Additionally, the interview data revealed that students did not transfer the algorithm to determine a specific value of A in terms of a specific value of w, Task 1, to the general case of expressing A in terms of w, Task 2. Analysis of the interview data further revealed that students who did not construct the formula, $l = 20 - w$ viewed the variables w and l as unknowns to be solved for rather than varying values that the width and length can assume. A variation view of variable, as discussed by Trigueros and Jacobs (2008), appears to be critical for viewing the problem goal as that of defining a function process to express the area of a rectangle with a perimeter of 40 feet in terms of the rectangle's width w. Analysis of the clinical interview data further revealed that an inability to understand the directive to express the area of a rectangle A in terms of its width w as a request to write "*A* = <some expression containing a *w*>" led to students not constructing a clear goal to guide their solution attempt. This study identified critical steps that students must engage with in order to successfully complete this task. Further research is necessary to determine how these ideas of variable, quantity, and function can be developed in curriculum as well as teaching practices so that more students are able to successfully complete and understand these function compositions in applied problems.

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