PAR FOR THE COURSE: DEVELOPING MATHEMATICAL AUTHORITY

Daniel L. Reinholz
University of California, Berkeley
reinholz@berkeley.edu

Perceived mathematical authority plays an important role in how students engage in mathematical interactions, and ultimately how they learn mathematics. This paper elaborates the concept of mathematical authority (Engle, 2011) by introducing two concepts: scope and relationality. This elaborated view is applied to a number of peer-interactions in a specialized peer-assessment context. In this context, self-perceived authority influenced the way feedback was framed (as either questions or assertions).

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Introduction

Individuals who are proficient in mathematics need not rely on an external authority; using the logic of the discipline, mathematicians *know* when they are right. Helping students become mathematicians requires helping them develop such *authority* grounded in mathematical reasoning. In this paper I build on the concept of mathematical authority (Engle, 2011) in the context of undergraduate mathematics education.

According to Engle (2011) authority begins to develop when students are “authorized” to share what they “really” think, and is solidified when students develop into local “authorities,” based on how students are positioned in the social space of the classroom. Implicit is the idea that authority “belongs” to students, existing beyond the confines of a single classroom situation. In what follows, I address the context-dependent nature of authority, and how it may (or may not) be transferred between contexts.

While authority does have some bearing on how one solves problems individually, it becomes most relevant when we think of one’s mathematical interactions with others; mathematical authority is a relational construct. Moreover, as is the case with self-efficacy (Bandura, 1997), it is an individual’s *perceived* mathematical authority that determines how they engage in mathematical interactions, not their *actual* ability to make authoritative statements about mathematics. Thus, it is important to consider authority not as a static attribute, but rather as something that depends on how an individual situates oneself (and is situated by others) in a mathematical interaction (Boaler & Greeno, 2000).

To extend the notion of mathematical authority, I introduce two concepts: scope and relationality. Perceptions of authority operate at a number of levels, increasing in generality (scope): (1) a specific problem, (2) a topic, (3) a mathematical domain, (4) the domain of mathematics, and perhaps (5) academics in general. Generalized authority is more readily transferable than localized authority, but is also less robust. (While an individual with a PhD in analysis has generalized authority that would extend to all of mathematics, a few failures in abstract algebra would be more detrimental to local self-perceived authority than a few failures in a related area of expertise (e.g., Banach spaces), because of the individual’s topic-specific - and more robust - authority in analysis.)

The relational nature of mathematical authority plays out in interactions between multiple individuals (e.g., partner work, small-group work, or a class discussion). Relationally, mathematical authority depends on: (1) one’s self-perception of one’s own authority, and (2) one's perception of another's authority. While scope is related to how authority transfers *between* domains or topics, relationality determines how authority manifests *within* a domain or topic. In general, individuals will act as though they have greater authority in situations where they perceive others to have less authority. For instance, a mathematics student would be much more likely to point out a (perceived) mathematical error to a fellow classmate than to a teacher, because, relatively speaking, she sees herself as having greater *relative* authority compared to the other student than to the teacher. In other words, the student is more likely to "trust" herself when challenged by another student than the teacher, because of the relative perceptions of authority.

There are notable exceptions to this generality though, particularly for individuals with low or high (general) self-perceptions of mathematical authority. For individuals with low perceived authority, they will act as though they lack authority regardless of the perceptions of other individuals. Even if such an individual perceives others to have *even less* authority, because the individual's self-perceived authority is so low, she still will not act as though she has authority. Moreover, individuals often have limited knowledge of the mathematical expertise of others, and thus must make situate themselves based on assumptions about others. Individuals with low levels of self-perceived authority will likely assume themselves to have lesser authority than their peers.

In contrast, individuals with a high-level of self-perceived authority may be willing to challenge the authority of others even if they perceive the other individual to have greater general mathematical authority. For instance, a competent (and confident) student in mathematics may be willing to challenge the authority of someone who is perceived as having greater general authority (e.g., a teacher, or successful competitive mathematician), because she has great enough self-perceived authority that she is willing to trust her own mathematical deductions (at least in this particular topic or problem). One of the reasons that it becomes possible for the student to challenge an individual with greater perceived authority is that the challenge is with respect to a specific problem or topic (i.e. limited in scope), and not the domain of mathematics overall (which is more general). In this way, it is possible for the teacher's mistake in a given problem (localized in context) to remain consistent with her overall perceived (general) authority.

Engle (2011) describes a hypothesized trajectory for developing mathematical authority: (1) learners are authorized to share what they think, (2) are recognized as authors of those ideas, (3) become contributors to the ideas of others, and (4) develop gradually into authorities about something. Because authority is a relational construct, it makes sense that it would develop out of social interactions. As students are recognized as authors of their own work and contributors to the work of others, their authority in that context is increased. However, unless this burgeoning sense of authority is reinforced in other contexts, it is likely that individuals will attribute their authority to the nature of that context, rather than attributing it to themselves, and thus not internalize authority.

In general, authority emerges from developing expertise with a specific set of problems or topics. For instance, one might perceive oneself as an authority on differentiation with the chain rule. If authority remains localized to this specific context, then it is unlikely that the individual will develop generalized mathematical authority. However, if the individual also develops authority in integration, epsilon-delta proofs, etc., then the individual will begin to develop authority in the domain of calculus. Developing authority in other mathematics classes as well will result in further generalization. This emphasizes the need for students to develop authority in a broad scope of activities, if it is to be internalized, generalized, and become transferable.

Having elaborated the concept of authority, I now consider a specialized peer-assessment activity designed to help students internalize mathematical authority.

Method

This paper draws from student interactions during the first six weeks of an ongoing study in introductory college-level calculus ($N = 53$). The larger study is focused on helping students develop skills of explanation and reflection. Students completed daily reflections, consisting of a self-rating (0 to 100%) of perceived understanding of the day's lesson and two other prompts. These self-ratings of understanding were averaged over the first 6 weeks of the course.

Students also engaged in a weekly peer-assessment and reflection (PAR) activity, sequenced in four steps. Students: (1) completed the PAR problem (individually), (2) self-assessed their understanding, (3) traded with a peer and gave peer-feedback during class, and (4) revised and turned in a final solution. The PAR activity helped students develop authority by allowing them to be authors of mathematics (in solving nontrivial problems) and contributing to the work of others through peer-assessment and feedback (cf. Engle, 2011). The PAR activity was framed such that students need not perceive themselves as experts in order to engage (e.g., rather than assessing "right" and "wrong," students provided feedback about what they understood and didn't understand in the solution; c.f. Reinholz, in press). While it is not the focus of this paper, the PAR activity requires students to be mathematically accountable to their assertions, in order to prevent students from developing unbounded (and unwarranted) authority (Engle, 2011).

Results and Analysis

To illustrate the role of perceived authority in peer-interactions, I present three cases of students with varying levels of self-perceived authority. Students' self-assessments of understanding in their daily reflections were used as a proxy for self-perceived authority (i.e. a student who consistently rated a high-level of understanding would be said to have a high-level of perceived authority, while a student who rates high levels of understanding for some topics would be said to have high-levels of authority *locally* for those topics). Although students were chosen based on their self-assessments, at least in this sample, self-assessments corresponded with actual performance on Exam 01. For this short paper, I focus on how scope of authority impacted peer-interactions.

Student 1: Low self-perceptions of authority (average 76.25% self-rated understanding)

Student 1 consistently rated low levels of understanding, which corresponded with the types of feedback he gave. All feedback was phrased as questions, even when it appeared the student was commenting on a perceived error in his peer's solutions:

- PAR01: Why was the second graph parabolic?
- PAR02: How did you know which equations to use for #2?
- PAR03: I'm unsure about #2 because what if the x can be canceled out?
- PAR04: For 2C why does the rate go down at the beginning?
- PAR05: In part (d) what is "it" referring to in your answer?

In PAR03, PAR04, and PAR05, the student gave critical feedback to the peer (in PAR01 and PAR02 the feedback appeared to be genuine questions, not critical feedback); in PAR03 he noticed an issue with the example his peer gave, in PAR04 he noticed that the graph was inaccurate, and in PAR05 the feedback related to the clarity of the explanation. Yet, all feedback was given in the form of questions (e.g., "did you think about X?") rather than as assertions. This feedback style reflects the student's low levels of self-perceived authority. Rather than making assertions that something might be incorrect, the student formulates questions that less directly challenge the peer's authority.

Student 2: High self-perceptions of authority (average 97.5% self-rating)
In contrast, student 2 had high global levels of self-rated understanding. This student's feedback became progressively more authoritative as the semester progressed (i.e. feedback was initially phrased as questions and over time became phrased as assertions):

- PAR01: Why aren't your axes labeled correctly?
- PAR02: Why does a line with a hole work for number 1?
- PAR03: 0/0 is indeterminant; why can't it equal 1 if you do the algebra?
- PAR04: Check where you have corners; corners are not differentiable.
- PAR05: There is a problem with the limit as x approaches a; if you can't find f(a) then the function doesn't exist so it can't be differentiable

While the feedback given by the student during the first 3 weeks seems clearly to be evaluative (e.g., why aren't your axes labeled correctly? is hardly a genuine question), it is not until later in the semester that the student began phrasing the feedback as assertions, as in PAR04 and PAR05. The increasing authority with which this student gives feedback corresponds with the student's continual experiences of success, which reinforced self-perceived authority (i.e. each time the student rated high levels of understanding and received teacher-feedback of success, authority was increased).

Student 3: Mixed self-perceptions of authority (average 92.5% self-rating)
Student 3 also appears to have a high level of self-perceived understanding, but if we look at the student's self-assessments for each of the PAR problems, we see that the perceptions of authority are localized in scope (in contrast to the other students whose self-assessments in their daily reflections were consistent with self-assessments in PAR). Student 3 gave the following peer-feedback:

- PAR01: I believe the x-intercept is wrong, as in the radius. The radius of the wheel will minimize at 1cm, not at 0cm.
- PAR02: Does rounding always work?
- PAR03: Why only graphs? Why no algebra with your written responses?
- PAR04: Explaining your process would really help in #2.
- PAR05: Could the quadratic function be more distinguishable?

Looking at this student's feedback, PAR01 and PAR04 were both stated authoritatively (as assertions rather than questions), but the feedback on other problems was stated as questions. There is no consistent feedback pattern as with the other students. However, when we look at the student's self-assessments for these particular problems we gain further insight in this seeming inconsistency (the PAR05 self-assessment was left blank):

- PAR01: I didn't notice any errors, and my assumptions made came from the information given, so they were justified; my answer makes sense in reality.
- PAR02: I am not sure if it was solved correctly. I have an assumption that "L" was the presumed limit, but I am not sure. My whole work was based on this, so if my assumption is wrong all of my work is wrong.
- PAR03: I checked my answers with those I could find in the textbook, and they all seemed to match up. The straightforward approach I used seems to be useful, never assuming that the answer you find is what you initially expect.
- PAR04: I think it was solved correctly. Logically, the graphs I created matched the actual physical behavior of the problem.

The self-assessments for PAR01 and PAR04 indicate the student felt confident in his understanding, which was reflected in his authoritative feedback. In contrast, the self-assessments for PAR02 and PAR03 were less confident (e.g., "I'm not sure if it was solved correctly," and "I checked my answers with those I could find in the textbook," which was evidently a subset of the actual answers given). For student 3 the high-level of general self-perceived authority seemed to be trumped by low perceptions of authority in a localized context (such as a single problem).

Discussion and Conclusions

Perceived mathematical authority influences how an individual acts in a situation independently of that individual's actual understanding. Perceived authority is both relational, dependent on an individual's relative perceptions of authority between individuals in a situation (relationality), and varies in scope (from local to more general). I presented a number of examples illustrating how the scope of self-perceived authority influences students' engagement in a specialized peer-assessment context. In particular, students who lacked authority in a given context tended to give less authoritative feedback (phrased as questions rather than assertions), even if their overall self-perception of authority appeared to be high.

Based on the above examples, self-perceived mathematical authority appears to have a profound impact on the types of feedback students give in a peer-assessment context. Students with higher perceived authority were more likely to give feedback in the form of assertions, while students with lower perceived authority tended to leave feedback in the form of questions, which are less likely to result in a challenge to the peer's authority. While students gave evaluative feedback and suggestions regardless of their perceived authority, the way in which feedback was framed was different. Although it is out of the scope of the present paper, the way in which feedback is framed is likely to influence whether and how other students respond to it. Students who are overconfident may mistakenly give overly authoritative feedback that could mislead other students, while students who are underconfident may lack leave feedback that is ignored because other students perceive it as too hypothetical or uncertain.

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