Abstract: At a research university near the east coast, researchers have restructured a College Algebra course by formatting the course into two large lectures a week, an active recitation size laboratory class once a week, and an extra day devoted to active group work called Supplemental Practice (SP). SP was added as an extra day of class where the SP leader has students work in groups on a worksheet of examples and problems, based off of worked-example research, that were covered in the previous week’s class material. Two sections of the course were randomly chosen to be the experimental group and the other section was the control group. The experimental group was given the SP worksheets and the control group was given a question-and-answer session. The experimental group significantly outperformed the control on a variety of components in the course, particularly when the number of SP days was analyzed.

Keywords: Cognitive Science, Interactive Compensatory Model of Learning (ICML), Worked Examples, College Algebra, Large Lecture.

INTRODUCTION

A Commitment to America's Future: Responding to the Crisis in Mathematics and Science Education states that “nationally 22% of all college freshman fail to meet the performance levels required for entry level mathematics courses and must begin their college experience in remedial courses” (2005, p. 6). The enrollment in College Algebra has grown recently to the point that nationally, there are estimated 650,000 to 750,000 students per year (Haver, 2007) and has surpassed the enrollment in Calculus. As a consequence, it is estimated conservatively that 45% of these students fail to receive a grade of A, B, or C and can reach percentages in the sixties at some colleges. This non-success has a dramatic effect on the transition to Calculus, let alone the transition from high school to college mathematics. To address this non-success of students at a large research university in the eastern part of the United States, faculty members teaching College Algebra have implemented a new structure in the course that emphasizes active learning through a day called Supplemental Practice, denoted SP.

THEORETICAL FRAMEWORK

The Interactive, Compensatory Model of Learning (ICML) provides the framework for understanding and improving classroom learning (see Figure 1). Schraw and Brooks (1999) refer to a wide range of literature that reinforces ICML. There are five main components of ICML: cognitive ability, knowledge, metacognition, strategies, and motivation, which affect learning. Brief definitions of the five main components of ICML can be found in (Miller and Schraeder, 2011) and more detail discussion in (Schraw and Brooks, 1999). Figure 1 shows that knowledge, metacognition, and strategies are so closely connected that they are combined together into one area in the figure. We will refer to this one area as the knowledge-regulation component. The ICML captures the interactions between these four components that affect learning and describes how one component can compensate for deficiencies in others. Each component can affect learning either directly or indirectly. For example, cognitive ability is related to learning directly, but also indirectly through knowledge-regulation. From Figure 1, one can see that each component directly affects
learning (the arrows from each component to learning) while only some components affect learning indirectly through another component. For example, motivation indirectly affects learning through knowledge-regulation, but not through cognitive ability. The numbers in the figure refer to the estimated correlation coefficient between two components. Each correlation coefficient is the estimated value of what has been measured in a number of empirical studies. Cognitive ability is correlated to learning with correlation coefficients ranging from 0.3 to 0.4 (Brody, 1992) and hence, the correlation coefficient of 0.3 between these components. The other correlation coefficients are shown in Figure 1. Schraw and Brooks (1999) state that “most experts agree that knowledge and regulation exert a strong direct effect on learning that is greater than the effects of either ability or motivational beliefs” (p. 9).

![Interactive, Compensatory Model of Learning](image)

**Figure 1: Interactive, Compensatory Model of Learning**

The compensatory part of the model refers to how students can compensate for a weakness in one component with another stronger component. For example, students who have weaker cognitive abilities can compensate by a stronger knowledge-regulation component. Through this iterative process, as they go from one topic to another topic in the course to gain knowledge, they more successfully compensate for their lower cognitive ability than other students. The notion of compensatory processes is supported by many different theories (Gardner, 1983; Perkins, 1987; Sternberg, 1994). Schraw and Brooks (1999) state the following compensation can occur: (1) ability compensates, in part, for knowledge and regulation, (2) regulation compensates for cognitive ability and motivation, and (3) motivation compensates for ability, knowledge and regulation.

**LITERATURE REVIEW**

The discipline of cognitive science deals with the mental processes of learning, memory, and problem solving. Worked Example research was developed from Seller’s cognitive load theory (1988). The total load on working memory at any moment in time is referred as the cognitive load. Most people can retain about seven “chunks” of information in their working memory and when they exceed that limit at any moment in time, there will be a loss of information in the working memory. In other words, there is an overflow of information in the working memory and cognitive overload. Cognitive overload can be thwarted if one limits information so that it does not exceed the students’ working memory. One way this can be done is to transfer information from working memory to long-term memory as information is being processed (or soon after). According to Sweller (1988), optimum learning occurs in humans when one minimizes the load on working memory, which in turn facilitates changes in long-term memory.

It has been suggested that worked examples reduce the cognitive load on a student and might optimize schema acquisition (Sweller and Owen, 1989; Sweller and Cooper, 1985). In addition, worked examples have been researched (and used) in a variety of subjects: mathematics (Cooper and Sweller, 1985; Zhu and Simon, 1987), engineering (Chi et al., 1989), physics (Ward and Sweller, 1990), computer science (Catrambone and Yuasa, 2006),
chemistry (Crippen and Boyd, 2007), and education (Hilbert, Schworm, and Renkl, 2004). Past research on worked examples in mathematics has been conducted in a laboratory setting. This research is conducted in a large-lecture classroom setting and concentrates on determining if worked examples helped promote success in the course. In addition, past worked-example research in mathematics has not dealt with college mathematics courses, classes in a large-lecture setting, or implementing an extra day of class to focus on working with students to master material. The research could be valuable to other researchers that are working to promote student success in large-lecture classes. The research questions that will be addressed in this study are “Do students in the experimental group that attend the majority of SP sessions earn significantly different course grades/exam scores/quiz scores/etc. than other students in the control group?” and “What if we look at the students that attend all of the SP sessions?”

METHODOLOGY

Course and Worked Example Worksheets

The setting for the research was a large-lecture 4-day College Algebra course with an annual enrollment of around 1000 students. This course is one of three different types of College Algebra courses at the university. One type of College Algebra is called the 3-day large-lecture College Algebra course that comprises of two lectures a week in a large-lecture setting and one day a week in the lab where students actively work in smaller-group math labs. The second type is the 4-day College Algebra course which has the same format as the 3-day College Algebra course, except the 4th day is spent in SP. The final type is a 5-day College Algebra course that is comprised of 5 lectures a week in a class size of approximately 40 students. The 5-day College Algebra class takes all quizzes and exams by pencil and paper and does not have a laboratory component. All College Algebra courses require specific placement exams scores. The 3-day College Algebra course requires the highest placement score and the 5-day College Algebra course requires the lowest placement score. In addition, students in the 3-day and 4-day large-lecture College Algebra courses take their computerized exams during lab and their quizzes at home on a computer. The pencil-and-paper labs are completed during lab time one day a week with the help of a java-based applet grapher.

During the SP days, worked-example worksheets were handed out to the students to work on in groups. Since the class was still in the large-lecture classroom setting with theatre style seating structure, students formed groups with other students near them as they saw fit. Usually, students worked with 1 to 3 other students seated close to them. The worked-example worksheets consisted of an expert solution of a College Algebra problem followed by a problem for the students to work out. An example of several worked-out examples from Worksheets is shown in (Miller and Schraeder, 2011) to give the reader some idea of the structure of the worked-example worksheets. The worksheet is always given to the students as one sheet (front and back) in a two-column format with headings on all worked examples, followed by the section in the textbook (Sullivan and Sullivan, 2006) that can be referenced later outside of class. There are approximately 8 to 10 worked examples and problems on each worksheet. The material on the worksheets consisted of some of the material covered during the previous weeks’ lecture (too much material to cover it all). No new material was ever covered and the worksheets comprised of problems directly from or derived from the problems in the textbook. The worksheets were never developed while referencing material from exams, quizzes, or labs. However, most of the questions from the exams and quizzes were similar to the homework in the book. Finally, the worksheets are modeled after worked-example research since it presents an expert’s solution to a problem followed by a problem for the student to work out. The only difference is that it is not plausible to ask the students to
not reference the worked example while working another problem and so this was never done. Furthermore, most studies on worked examples state that the student should be given a similar problem (very similar in some cases), but in SP, the problems students were asked to do vary from very similar to somewhat different.

**Experiment**

The researcher randomly designated one of the course sections as the control group (n = 177) and the other two sections as the experimental group (n = 320). In the experimental group, the students were given a “worked-out example” worksheet at the beginning of each of the 13 SP days and asked to work in groups to complete the worksheet. Three to four class assistants circulated around the room to answer any student questions about the worksheet. In the control group, a graduate student organized a question-and-answer session during the extra day instead of giving a worksheet to the students. Students were able to get any question answered, but the graduate student only answered student questions and did not generate questions herself. Students in the experimental group who attended 8+ SP days were grouped into the experimental group we will call EXP8, and students in the experimental group who attended all 13 SP days were grouped into the experimental group we will call EXP13. Quantitative data (course scores on exams and quizzes, SP days attended, class attendance, total points,…) was collected for each student in both the control and EXP8 and EXP13 groups and analyzed at the end of the semester.

**Data**

We present the data for two experimental groups below. The following components are analyzed with the following breakdown in the course: Current (Total) Points – 1000, Current Points without attendance – 900, Current Points without attendance and Labs – 700, Current Points – Exams Only – 600, Test 1 through 4 – 100, Quizzes – 100, and Final Exam – 200.

**The EXP 8 versus Control Group**

Data from the experimental and control groups were compared on a variety of levels by using t-test with equal and unequal variances depending on the data. The experimental and control group had similar levels of retention (number of students that completed the course) at 80.5% and 84%, respectively. At the beginning of the semester, all students were given an old ACT math test that consisted of 60 questions. Students were given extra credit points for the ACT exam on a sliding scale. This ensured that the better a student performed on the exam, the more extra credit (up to 10 points) he/she earned. The exam gave a good measure of students’ prior mathematical knowledge. Figure 2 shows the control and EXP8 groups’ mean scores of 28.40 and 26.88 with standard deviations of 6.41 and 6.91, respectively. The control group significantly outperformed the EXP8 (p=0.01) on the pre-ACT exam.

![Prior/Post Mathematical Knowledge](image)

**Figure 2: Prior/Post Mathematical Knowledge for Both Groups**

At the end of the semester, students were given the same ACT exam to measure their post mathematical knowledge. Figure 2 shows the control and EXP8 groups earned a mean ACT score of 32.81 and 32.35, with standard deviation of 6.46 and 7.13, respectively. There was no significant difference between the mean ACT scores of the two groups. We note the EXP8 improved to a point that they were comparable with the control group on the post-ACT.
The data for total points in the course (Current Points), total points without attendance (CP w/o Attend), total points without attendance or labs (CP w/o Attend, Labs), and Current points for just exams (CP Tests Only), were compared between the groups. Figure 3 shows the current points for the two groups and Table 1 shows the mean current points with standard deviations.

Figure 3: Total Points in the Course for Both Groups

<table>
<thead>
<tr>
<th></th>
<th>Current Points</th>
<th>CP w/o Attend</th>
<th>CP w/o Attend, Lab</th>
<th>CP Tests Only</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control Gp</td>
<td>698.81</td>
<td>605.88</td>
<td>453.24</td>
<td>381.72</td>
</tr>
<tr>
<td>(n=177)</td>
<td>(150.51)</td>
<td>(140.25)</td>
<td>(120.04)</td>
<td>(105.48)</td>
</tr>
<tr>
<td>Exp8 Gp</td>
<td>750.15</td>
<td>625.37</td>
<td>470.82</td>
<td>396.25</td>
</tr>
<tr>
<td>(n=279)</td>
<td>(117.00)</td>
<td>(113.88)</td>
<td>(104.10)</td>
<td>(91.69)</td>
</tr>
</tbody>
</table>

Table 1: Means for Total Points in the Course for Both Groups and Standard Deviations

There were strong significant differences between the mean scores of the control and EXP8 with respect to all Current Points (Total Points): Current Points (p = 0.00007), CP w/o Attend (p = 0.00019), CP w/o Attend & Lab (p=0.00029), and CP Tests Only (p = 0.00059). Note that current points did not include any extra credit (i.e. pre/post ACT exam).

The two groups were compared with respect to each exam, the final, and quizzes. Figure 4 shows the two groups mean scores on each exam, the final, and quizzes and Table 2 shows the exact scores and standard deviation in parentheses.

Figure 4: Tests and Quizzes for Control and Experimental Group

<table>
<thead>
<tr>
<th></th>
<th>Test 1</th>
<th>Test 2</th>
<th>Test 3</th>
<th>Test 4</th>
<th>Final</th>
<th>Quizzes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control Gp</td>
<td>68.84</td>
<td>66.86</td>
<td>66.58</td>
<td>67.23</td>
<td>56.10</td>
<td>71.52</td>
</tr>
<tr>
<td>(n=177)</td>
<td>(16.21)</td>
<td>(19.93)</td>
<td>(19.61)</td>
<td>(22.67)</td>
<td>(24.02)</td>
<td>(18.89)</td>
</tr>
<tr>
<td>Exp8 Gp</td>
<td>68.62</td>
<td>70.86</td>
<td>72.24</td>
<td>72.35</td>
<td>64.</td>
<td>78.09</td>
</tr>
<tr>
<td>(n=279)</td>
<td>(15.97)</td>
<td>(17.03)</td>
<td>(17.81)</td>
<td>(20.02)</td>
<td>61(20.04)</td>
<td>17.62</td>
</tr>
</tbody>
</table>

Table 2: Means and Standard Deviations for Test 1 through Test 4, Final and Quizzes

The EXP8 significantly outperformed the control group on every test, except Test 1: Test 2 (p = 0.011), Test 3 (p=0.00079), Test 4 (p = 0.0060), Quizzes (p = 0.00012), and Final Exam (p = 0.000054). There was no difference between the control and EXP8 with respect to Test 1.

Course G.P.A. was calculated to compare the two groups on the average course grade earned. This was accomplished by assigned a quantitative score for the final grade that each student earned in the course (A=4, B=3, C=2, D=1, and F=0). Figure 5 shows the course grade point average for control group (1.97) and EXP8 (2.34) with standard deviations 1.17 and 1.16, respectively. The EXP8 had a significantly better course grade point average than the control group (p = 0.00048).

Course G.P.A.
EXP8 versus EXP13

The ICML supports that motivation is an important part of a students learning. In terms of this study, we examined the students that attended all of SP sessions versus the students that attend more than half of the SP sessions. We examined this in the study because of the ICML and the fact that students voluntarily attend the SP days. Therefore, we are determining student success in the course when SP days are factored into the analysis. Instead of displaying graphs, we will state the means, standard deviations, and p-values in the table for the EXP13 and the EXP8.

Table 3 shows that the EXP13 significantly outperformed (in most cases very strongly) the EXP8 in every facet of the course. Therefore, the students that were motivated to attend all the SP sessions significantly outperformed the students that were motivated to attend 8 or more SP sessions. The differences would be greater if we only grouped the students that attended 8 to 12 SP sessions.

<table>
<thead>
<tr>
<th></th>
<th>Mean for EXP8 (n=188)</th>
<th>Mean for EXP13 (n=297)</th>
<th>Standard Deviation For EXP8</th>
<th>Standard Deviation for EXP13</th>
<th>P – Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current Points</td>
<td>750.15</td>
<td>793.74</td>
<td>116.99</td>
<td>85.91</td>
<td>0.00003</td>
</tr>
<tr>
<td>CP w/o Attend</td>
<td>652.69</td>
<td>693.99</td>
<td>113.88</td>
<td>85.59</td>
<td>0.00005</td>
</tr>
<tr>
<td>CP w/o Attend &amp; Lab</td>
<td>491.37</td>
<td>528.28</td>
<td>104.10</td>
<td>75.70</td>
<td>0.00005</td>
</tr>
<tr>
<td>CP Tests Only</td>
<td>413.28</td>
<td>443.21</td>
<td>91.69</td>
<td>69.71</td>
<td>0.00024</td>
</tr>
<tr>
<td>Test 1</td>
<td>68.62</td>
<td>73.59</td>
<td>15.97</td>
<td>13.61</td>
<td>0.00093</td>
</tr>
<tr>
<td>Test 2</td>
<td>70.86</td>
<td>74.36</td>
<td>17.03</td>
<td>15.39</td>
<td>0.02337</td>
</tr>
<tr>
<td>Test 3</td>
<td>72.24</td>
<td>78.25</td>
<td>17.82</td>
<td>14.66</td>
<td>0.00029</td>
</tr>
<tr>
<td>Test 4</td>
<td>72.35</td>
<td>79.49</td>
<td>20.02</td>
<td>13.36</td>
<td>0.00002</td>
</tr>
<tr>
<td>Final</td>
<td>129.29</td>
<td>137.52</td>
<td>40.15</td>
<td>32.72</td>
<td>0.01620</td>
</tr>
<tr>
<td>Quizzes</td>
<td>78.09</td>
<td>85.07</td>
<td>17.62</td>
<td>11.29</td>
<td>0.000002</td>
</tr>
<tr>
<td>Course GPA</td>
<td>2.344</td>
<td>2.752</td>
<td>1.16</td>
<td>0.955</td>
<td>0.00017</td>
</tr>
</tbody>
</table>

Table 3: Means, Standard Deviations, and p-value for 8+ and 13 Experimental Groups

RESULTS AND IMPLICATIONS TO TEACHING

According to the ICML, motivation plays an important part in learning. SP was implemented as an extra day where students could actively engage in their own learning using worked examples from cognitive science. This study has shown that not only does the experimental group significantly outperform the control group (Miller and Schraeder, 2011) in many of the measured course categories, but the students that are motivated to attend 8 or more SP sessions significantly outperform the control group in a very convincing fashion in the majority of these categories. In addition, the highly motivated students (EXP13) significantly outperform the EXP8 in a very strong fashion in every facet of the course. This provides strong evidence that motivation is a very important part in a student’s learning of College Algebra. In fact, the students in the EXP13 have varying levels of prior knowledge measured by the pre-ACT (low, middle, and high), but become very successful learners. That
is, students compensate for their weak prior knowledge with a stronger motivation component (along with the knowledge-regulation component) to become very successful in the course. In the EXP8 group, the middle prior knowledge group is the only group that significantly outperforms the control group similar to what (Miller and Schraeder, 2011) found for the experimental group versus the control group.

This study can provide some insight to teaching such as worked example worksheets (or other similar interventions grounded in research) imbedded into a class as active learning help students become more successful learners. In addition, the authors believe that the worksheets provide a basis that helps students when studying and working homework problems and quizzes which is backed up by student’s comments on an end of the semester survey. Furthermore, working to design components of the class that motivate students to learn is very important in a course. Since the low prior knowledge students, in general, struggle to be successful in the course, instructors could work on providing additional intervention (perhaps mandatory) so that students could compensate for this weak area along with strengthening their knowledge. The authors plan to work on developing an additional intervention for the low prior knowledge students so that they can become more successful in the course. The authors also plan to research the motivated groups in more depth and present more detailed data available on prior knowledge levels for the Control and Experimental Groups.

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