FORMATIVE ASSESSMENT AND STUDENTS' ZONE OF PROXIMAL DEVELOPMENT IN INTRODUCTORY CALCULUS

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One of the challenges of teaching introductory calculus is the large variance in student backgrounds. Formative assessment can be used to target which students need help, but little is known about why formative assessment is effective with adult learners. The purpose of this qualitative study was to investigate which functions of formative assessment as described by Black & William's 2009 framework help students progress through their Zone of Proximal Development. By regularly collecting information from low-stakes opportunities for students to demonstrate their current understanding, instructors were able to target subsequent class discussion on critical scaffolding for student growth. The formative assessments also enabled students to evaluate their own progress and ask clarifying questions and, provided students who would not ordinarily ask questions during class opportunities for legitimate peripheral participation.

Key words: approximation framework, formative assessment, self-monitoring, Zone of Proximal Development

Introduction

Formative assessments, low stakes assignments given to assess students' current level of understanding, increase student achievement (Black & Wiliam, 2009; Clark, 2011), but little is known about how implementing formative assessments facilitates this achievement gain. The purpose of this research was to study the impact of formative assessment on students' engagement in their Zone of Proximal Development (ZPD) in a calculus course designed with Oehrtman's (2008) approximation framework. Our central research question is: How does formative assessment impact students' engagement in their ZPD and conception of the limit structures as developed in Oehrtman's (2008) approximation framework for calculus instruction?

Understanding how the use of formative assessment affects college students' engagement in their ZPD and development of a particular conceptual structure can advance the theory of formative assessment, which has been most prominently influenced by research in European primary and secondary schools (Black & Wiliam, 1998; 2009). Black & Wiliam's (2009) framework of formative assessments suggests that there are five functions of formative assessment (Figure 1).

(1) Clearly communicating learning goals
(2) Allowing instruction to be based on students' current level of understanding
(3) Providing learners with feedback that scaffolds learning
(4) Giving peers a common experience for future collaboration
(5) Raising students' ownership of their learning process through increased metacognition

Figure 1. The five purposes of formative assessment

Theoretical Perspective and Methods

There are several characterizations of the ZPD (Vygotsky, 1987); this report will focus on the interplay between students' spontaneous and scientific concepts and scaffolding that supports students in deepening their conceptual understanding. Results from this study reveal ways in which formative assessment enabled instructors to better assess and target the areas in which critical scaffolding was needed and that this process increased both students' self-monitoring of their understanding and opportunities for peripheral participation in the classroom. Generally, the ZPD can be identified by determining what students can do but only with assistance. The learner is a peripheral participant in this assessment and subsequent scaffolding, because they are being assisted by a more central member of the learning community (Lave & Wenger, 1991; Smagorinsky, 1995). As the learner gains expertise, scaffolding may be reduced and the learner becomes a more central participant in the community of practice.

We recruited participants from classes utilizing Oehrtman's (2008) approximation framework as a coherent approach to instruction in introductory calculus. This framework is built upon developing systematic reasoning about conceptually accessible approximations and error analyses but mirroring the rigorous structure of formal limit definitions and arguments (Oehrtman, 2008, 2009). This study focused on the three multi-week labs developing the most central topics in the course: Lab 3 (limits), Lab 4 (derivatives), and Lab 7 (definite integrals).

This qualitative study centered on a document analysis (Patton, 2002). Our primary sources of data were student documents: formative assessments, homework assignments, and exams of all students in two sections of introductory calculus, with particular attention paid to ten students who each participated in at least one interview. The first author also observed the classrooms the day before and the day after the weekly formative assessment was distributed to the students and debriefed the instructors on a weekly basis to obtain their observations of student and classroom learning trajectories.

Figure 2 provides a portion of a typical formative assessment. These assignments were given prior to each lab (pre-lab) and after each day of lab work (post-lab). The first questions of our formative assessments were conceptual questions about important aspects of the approximation structures in the current lab (not shown in Figure 1). Two open-ended questions always appeared as the last two questions of every formative assessment. An analysis of students responses to the questions were used to plan a brief intervention in the next class addressing the problematic issues.

We coded the data chronologically. First, each action a student needed to take to successfully complete each pre-lab and final lab report was listed. Each student's assignment were then coded for each of these actions; we noted if the action, such as correctly identifying over-and underestimates, was present/absent or appropriate/inappropriate if an action was present. When coding the observation notes during labs we noted which points of difficulty groups asked for help on and made counts of how often those points of difficulty appeared in various groups. The post-labs were coded for three things: (1) mathematical errors students made on any calculational questions, (2) noting if the students identified the problems they had with calculations or parts of the lab accurately, and (3) coding all questions by the concept students found troubling. During the intervention, the first author observed the class using three minutes to count student behaviors (paying attention to the instructor, taking notes, texting or other off task behavior) and then spent three minutes recording impression. Those observation notes were coded for changes in participation patterns.

We recorded which concepts each student ($n = 46$) explicitly stated they did or did not understand and what, if any errors they made on computational questions on each formative assessment. Students' responses to the formative assessments were triangulated with field notes of the classes immediately before and after the lab, as well as their submitted lab reports. Each student's work was coded for particular areas of improvement after the intervention. This initial coding was then analyzed at three levels: by interview participants, by grade bands, and by assignment. At all levels we attempted to identify when a concept was a point of difficulty (entered the ZPD) and ceased to be a point of difficulty (left the ZPD)

Post-Lab 3a: Locate the Hole (Limits)
Directions: Answer the following questions to the best of your ability. Responses need not be lengthy, but should answer all parts of the question.

1. Which question is your group working on?
2. What have you figured out about the answer so far?
3. Write a short paragraph that answers the following two questions. What mathematical concepts or phrases used so far this week do you recognize from calculus? From other mathematics courses?
4. What questions do you have about the material we have covered so far in class?

Figure 2. A typical formative assessment

Findings

For the purposes of this paper, we will focus on Lab 4, the second of the three central approximation framework labs in the semester. During Lab 3, the first lab developing the elements and relationships of the approximation framework, students approximated the *y*-coordinate of a removable singularity in a given function. On the formative assessments, students were able to evaluate the function at nearby *x*-values to approximate to unknown *y*-coordinate with little difficulty. They struggled to use information about the monotonicity of the function to consistently identify over- and underestimates and to understand the difference between errors and error bounds. Students also needed significant scaffolding to draw graphs that were appropriately sized, scaled, and labeled to effectively represent the relevant quantities and relationships. On the formative assessments, students identified their difficulties with errors and error bounds, but not the other two areas mentioned. The scaffolding provided in class addressed all three points of difficulty, and students improved in all three areas from their post-lab to their final write-up. Students who earned B's or C's in the course showed the most improvement, which is consistent with the literature.

Although students submitted exemplary graphs for their final Lab 3 write-up and were given extensive instructions on how to construct high quality graphs for Lab 4, the graphs on pre-lab 4 were inappropriately small with little detail and labeling (Figure 3 is a typical example). The pre-labs allowed the instructor and undergraduate teaching assistants to immediately respond to students' need for additional assistance constructing their graphs. The other points of difficulty during the first day of lab 4 were the from applying context-specific concepts in lab 3 to lab 4; groups wanted to approximate slopes and identify over- and underestimates using the heuristic for Lab 3, using *y* values as approximations, rather than using average rates of change. On the post-lab that night, students repeated this mistake, but did not indicate any confusion; instead students asked about identifying over- and underestimates or the difference between errors and error bounds. The intervention in the next class discussed why *y*-values were not appropriate approximations, how to classify approximations, and the graphical, algebraic, and numerical representations of errors and error bounds.

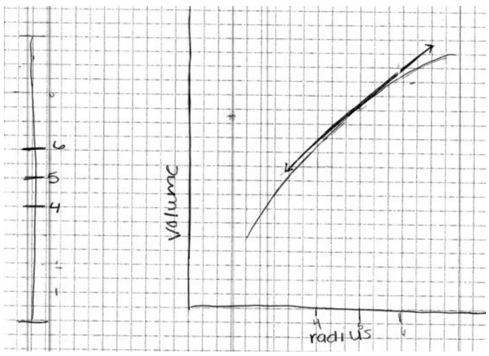

Figure 3. A typical pre-lab 4 graph

In the second week of Lab 4, students completed their problems with minimal assistance; although two groups in each class required additional help entering complicated functions into their calculator. The subsequent post-lab asked students to make connections between the algebraic and graphical representation of the derivative (Figure 4). Students had a median of 4 errors (out of 7 questions), but 66% of the students reported that they were sure some of their answers were wrong, evidence of self-monitoring.

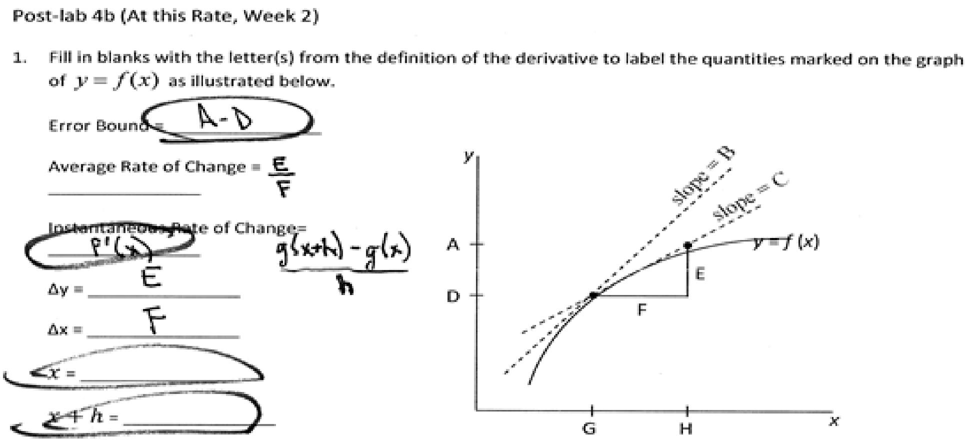

Figure 4. Post-lab 4b, Question 1

The formative pre-labs and post-labs allowed instructors to identify students' ZPD and provide the scaffolding they needed. Students were more likely to take notes, pay attention and refrain from texting in class during lab interventions than any other time in the class. Students were reasonably successful in identifying when they were making mistakes or did not understand a concept in their post-labs. For this lab, the second, third, and fifth functions of formative assessment were the ones that most identifiably helped students engage in their ZPD (Figure 1). In their formal lab reports, the only students who did not successfully improve their graphs, use the correct approximations, classify approximations, or distinguish errors and error bounds were those not in attendance during the instructor-led intervention. The students who did not complete post-labs but attended the intervention showed the same improvements as the students who completed post-labs, which suggests that their peripheral participation in the intervention was sufficient for students to progress through their ZPD.

Students showed the most improvements on their graphical representations, for example, the graph in Figure 5 was submitted by the same student who submitted the graph in Figure 2. On Lab 7, on definite integration, students turned in appropriately sized, scaled, and labeled graphs on their pre-lab, and the points of difficulty were all context-specific or calculator-based rather than difficulty with elements and relationships in the approximation framework, indicating significant progress since their difficulties with these issues in Lab 4.

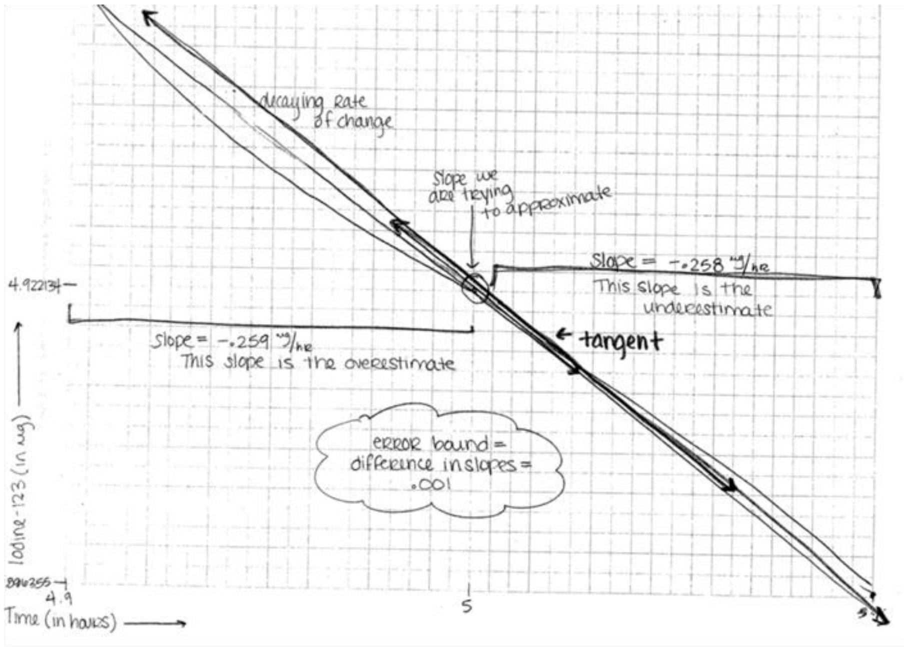

Figure 5. A typical final Lab 4 report graph

Discussion

While the formative assessments were intended to provide a snapshot of students' current understanding and allow instructors to make decisions on what scaffolding their class needed, the act of completing the formative assessment also helped students improve their self-monitoring skills and gave them opportunities to peripherally participate in class without becoming a central participant. Hence, the asynchronous formative assessments had both instructor-centered functions and student-centered functions. The pre- and post-labs gave the instructor a chance to evaluate students current understandings of the activity and target the scaffolding in the next class as precisely as possible. Students gained opportunities for ownership of the material through self-monitoring and peripheral participation opportunities. Although the completion rate of the formative pre- and post-labs was lower than for the labs themselves, students that did not complete formative assessments but attended the intervention still improved in the areas instructors scaffolded; this suggests that all students derived some benefit from the scaffolding based on the formative assessments. The next phase of analysis on this data will detail students' development aspects of the approximation framework from spontaneous concepts to scientific ones. Questions that we will pose to those attending our talk are: (1) What additional insights could be sought from analyzing interviews of students explaining their reasoning behind all of their written responses? (2) How might we more fully integrate multiple characterizations of the ZPD in our data analysis?

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