SELF-INQUIRY IN THE CONTEXT OF UNDERGRADUATE PROBLEM SOLVING

Todd A. Grundmeier
Dylan Retsek
Dara Stepanek

California Polytechnic State University, San Luis Obispo

Self-inquiry is the process of posing questions to oneself while solving a problem. The self-inquiry of thirteen undergraduate mathematics students was explored via structured interviews requiring the solution of both mathematical and non-mathematical problems. The students were asked to verbalize any thought or question that arose while they attempted to solve a mathematical problem and its nonmathematical logical equivalent. The thirteen students were volunteers who had each taken at least four upper division proof-based mathematics courses. Using transcripts of the interviews, a coding scheme for questions posed was developed and all questions were coded. While data analysis of the posed questions is ongoing, initial analysis suggests that the "good" mathematics students focus more questions on legitimizing their work and fewer questions on specification of the problem-solving task. Additionally, the self-inquiry of "fast" problem solvers mirrored that of the strong students with even less focus on specification questions.

Keywords: Problem solving, Proof, Self-Inquiry, Logic, Questioning

Many teachers refuse to simply answer a student's question; instead, these teachers insist on responding to the student's misconceptions with other related questions that the student can answer, slowly scaffolding the student's responses until the student has answered (knowingly or unknowingly) their own question. This method, when done correctly, allows the student to recollect related knowledge, receive a confidence boost in their own knowledge of the subject, and receive a lesson in problem-solving strategies that could be utilized to solve future problems. This method of answering questions with other questions seems to work extremely well for student ownership of material, but the question remains as to why students don't ask themselves some or all of these leading questions. Since the student is capable of answering the posed questions that lead them to the solution, what is stopping the student from posing these questions themselves? Is effective self-inquiry a mark of a "good" student? What types of questions do these "good" students ask themselves while problem solving? More importantly, how can we foster pedagogical knowledge from these "good" students' questions so that teachers can guide all students toward productive self-inquiry?

A goal of this study is to explore these questions by trying to document and code questions that students ask during the problem solving process. While related research has been conducted in secondary education and reading comprehension (Kramarski & Dudai, 2009; King, 1989) and in general mathematical thinking (Schoenfeld, 1992), it seems that the self-inquiry of undergraduates in the process of mathematical problem solving has not been explored. Therefore another goal of this project is to begin this line of inquiry and add to the current mathematics education research related to problem solving.

Thirteen mathematics majors were interviewed with the instruction to verbalize every thought process and question that arose as they attempted to solve two problems. The first was a problem in a mathematical context with contrived terminology that was new to all students:

Mathematical Context* (Pleasant Sets)**: A set *S* of real numbers is called ***pleasant if each element of *S* has both a unique immediate successor in *S* and a unique immediate predecessor in *S*.

Let *S* be a pleasant set. Suppose the numbers a, b, c, d, and e belong to *S* and satisfy

(i) b is greater than or equal to the successor of d and less than or equal to the predecessor of e;
(ii) a is the successor of d;
(iii) c is greater than or equal to the successor of e.

Put a, b, c, d, and e in numerical order.

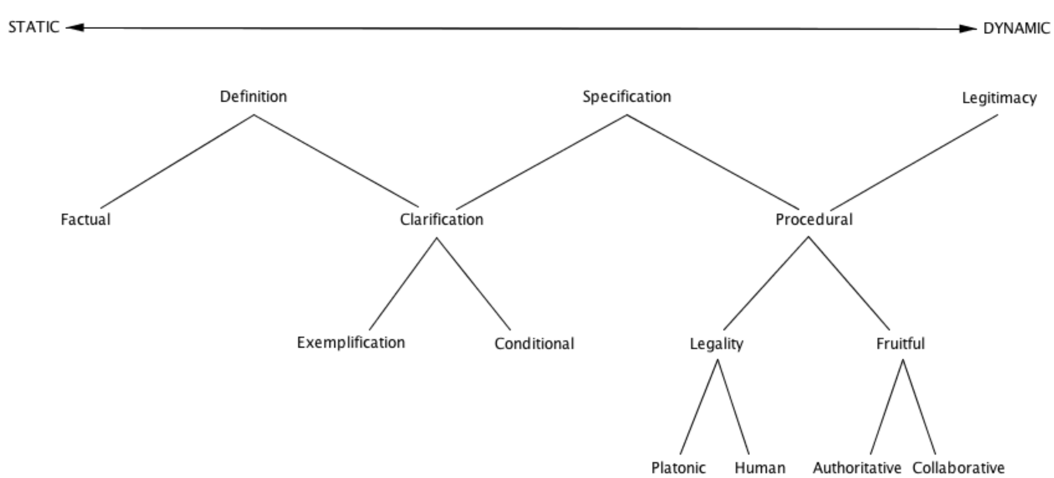
The second problem, though posed in a nonmathematical context, was logically equivalent to the first:

***Nonmathematical Context* (FeedingTime)**: Zookeeper Jane feeds the animals in 15 minute intervals every morning such that

(i) the giraffes are fed after the monkeys but before the zebras;
(ii) the bears are fed 15 minutes after the monkeys;
(iii) the lions are fed after the zebras.

What is the feeding schedule?

After completing the interviews the transcripts were analyzed by all three authors and the questions posed by each participant were agreed upon. This process led to a classification of the questions being posed during the interviews. The following question tree was developed to exhaust the coding of all questions posed. The question tree will be explained in detail during the presentation.



The authors then individually coded the posed questions using the question tree and met to discuss any discrepancies and agree on codes for all questions posed. In order to explore the self-inquiry of "good" students the authors defined the statistic RSQ (Relative Success Quotient) and calculated an RSQ for all students. To calculate the RSQ the authors focused on the 11 upper division courses that had been taken by at least 7 of the participants. For each course the average GPA and standard deviation of grades were calculated for the last 5 years of course offerings. A participant's RSQ is then calculated as the average number of standard deviations their grades are away from the mean for the courses they had completed from the 11 chosen. More details of the RSQ calculation will be described in the presentation.

Participants clearly fell into three RSQ categories, deemed low, middle and high, and data has been organized accordingly for the pleasant sets question.

	Low	**Middle**	**High**
RSQ	*-0.167*	*0.479*	*.879*
Average time to solve	0:12:55	0:15:16	0:06:01
Average # of Q's	13.75	16	9.2
Frequency of Q's	0:01:01	0:01:10	0:00:47
Definition Q's	29.09%	18.75%	28.26%
Specification Q's	29.09%	46.875%	26.09%
Legitimacy Q's	41.82%	34.38%	45.65%

It is interesting to note that the students with a high RSQ were more efficient problem solvers who asked fewer questions. More interesting, though, is that these fewer questions focused on legitimizing their problem solving efforts.

Similar data analysis was completed based on the time it took participants to solve the pleasant sets problem. Again participants fell into three groups deemed slow, medium and fast, and the table below depicts the data organized by these groups.

	Slow	**Medium**	**Fast**
Average time to solve	*0:21:02*	*0:10:07*	*0:04:13*
RSQ	0.117	.645	0.425
Average # of Q's	20.75	11.6	6
Frequency of Q's	0:01:01	0:00:52	0:00:42
Definition Q's	26.51%	22.41%	25%
Specification Q's	42.17%	36.21%	8.33%
Legitimacy Q's	31.33%	41.38%	66.67%

While the medium group had the highest average RSQ, the self-inquiry of the fast problem solvers was most similar to that of the participants with a high RSQ as the questions focused on definitions and legitimizing their work. It is interesting to note that the fast problem solvers posed a very low percentage of questions that served the purpose of specifying the problem-solving situation.

This preliminary report will focus on the development of the question tree used for coding and will present further data related to both the pleasant sets and feeding time examples. The goal of the presentation will be twofold. First, to address the questions posed at the beginning of this proposal with relation to this set of data. Second, we hope to receive feedback from the RUME community about the initial direction of this project and the questions below will be helpful in framing our conversation.

Questions

1. Is the question tree a useful tool for analyzing questioning in the context of mathematical problem solving?
2. Is RSQ the appropriate statistic for defining "good" students? Are there other options for defining "good" students that may shed a different light on the data?
3. What other options are there for research design to try to identify self-inquiry in undergraduates?
4. Is self-inquiry a topic of interest to the RUME community?
5. Should we expect different levels of self-inquiry throughout a math major's undergraduate career?

References:

Kramarski, B. and Dudai, V. (2009). Group-metacognitive support for online inquiry in mathematics with differentiated self-questioning. *Journal of Educational Computing Research, 40*(4), pgs. 377-404.

King, A. (1989). Effects of self-questioning training on college students' comprehension of lectures. *Contemporary Educational Psychology, 14*(4), pgs. 366-381.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense-making in mathematics. In D. Grouws (Ed.), Handbook for Research on Mathematics Teaching and Learning (pp. 334-370). New York: MacMillan.