GUIDED REFLECTIONS ON MATHEMATICAL TASKS: FOSTERING MKT IN COLLEGE GEOMETRY

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Abstract: *This study is a part of ongoing research on development of Mathematical Knowledge for Teaching (MKT) in mathematical content courses. Reflective practice represents a central theme in teacher education. The purpose of this reported study was to understand the role of guided reflections on mathematical tasks in a college geometry course. We were also interested in understanding how guided reflections on mathematical tasks would effect teachers' development of MKT. Our research data consist of participants' reflections, teaching scenarios, and pre-post test results. In this study we developed a workable framework for data analysis. Audience discussion will address questions related to the proposed analysis framework and development of MKT in college mathematics courses.*

Key Words: [College Geometry, Mathematical Knowledge for Teaching, Reflections, Framework, Pedagogical Content Knowledge]

Theoretical Background

Recently there has been high interest in the knowledge of secondary mathematics teachers (Chamberlin, 2009; Proulx, 2008; Hough, O'Rode, Terman & Weissglass, 2007). In the following study, we look at the role of reflections in mathematics teachers' knowledge development. Various studies documented a connection between reflection on mathematical experiences and an increase in mathematical knowledge (Burk & Littleton, 1995; Chamberlin, 2009; Bjuland, 2004; Wheatley, 1992). Specifically, our study focuses on the effect of reflections on Mathematical Knowledge for Teaching (MKT). Much work has been done on understanding MKT in elementary mathematics (Ball, 1991; Ball, Hill, & Bass, 2005, Hill & Ball, 2004). Our study is situated in college geometry.

In 2004, Hill, Ball, and Schilling developed a framework for characterizing teachers' MKT consisting of two areas, Pedagogical Content Knowledge (PCK) and Subject Matter Knowledge (SMK). Our interest in this study is to observe secondary mathematics teachers' growth in MKT by assessing their change in PCK and SMK and the quality of their reflections on mathematical tasks. In this study, we attempt to answer the following research questions:

1) In what way do reflections on mathematical tasks affect the growth of MKT in secondary mathematics teachers?
2) Do the type and quality of the reflections have an effect on what kind of growth (SMK or PCK) the teachers experience?

Data Sources

The preliminary report is based on early analysis of participants' reflections, teaching scenarios, and pre-post test results. During the summer of 2011, 18 students enrolled in a *Teaching Geometry from Problem Solving to Proving* course at a small northeastern liberal arts college. The content of this course included a research-based curriculum, which was designed as a part of an MSP Grant. This task-based course (see Appendix A for an example of a task used in the course) was designed around problem-solving episodes, where the students would engage in investigating mathematical concepts through a reflective process; the objective was to increase the knowledge of mathematics while providing frequent opportunities to think about how they would teach the mathematical concepts being investigated. The course focused on Euclidean Geometry, including some fundamental concepts such as congruence, similarity, construction, area, etc. To measure the change in MKT, we administered a pre-test before the course, and a post-test (exactly the same content) after the course. As a part of formative course assessment, students solved and designed a variety of mathematical problems, along with reflections on their learning experiences. The data collected included participants' reflections, teaching scenarios, and pre-post test results. Typical course activities were multifaceted, targeting several domains of teachers' knowledge, in a coherent and interconnected manner.

Research Methodology

Designing the Framework: Reflections

The research methodology for this project was developed on Chamberlain's original work (2009). Originally, we assessed the participants' reflections in four categories: identification of the purpose of the task, recognition of cognitive difficulties that their students might have when trying to complete the task, situation of the task in their teaching by acknowledging where in the curriculum the task would fit or what the appropriate grade level for which the task should be utilized, and finally identification of the pedagogical strategies that could be used to teach the task. However, we decided to hone in on the first category, identification of purpose, because we noticed that there were two different types of purpose being recognized by the participants: mathematical and instructional.

Thus, we defined each type of purpose, differentiating from strong to weak within these purposes and continued our research through this scope (See Table 1 in Appendix A). A weak mathematical purpose is one that generalizes the steps taken to solve the problems or states what mathematical concept is being addressed in the given task. A strong mathematical purpose recognizes connections to mathematical contexts not directly used in the task. A weak instructional purpose included a general comment on the strategy used to complete the task. A strong instructional purpose provided connections made to teaching outside of the specific task and took the students into consideration. Since mathematical and instructional ideas are not completely separate, there were some participants that identified both types of purposes. Their responses were placed into both classifications and assessed according to the strength within those categories. Examples of these comments are included in the results portion of this paper and are useful in clarifying the meaning of each category. After finalizing the rubric, we read each of the reflections and scored them according to the rubric.

Designing the Framework: Pre- and Post-Tests

At the stage of assessing the change of subject matter knowledge and pedagogical content knowledge of each of the teachers, we decided to analyze the results of the pre- and post-tests.

Looking at particular subsets of all the questions answered on the test, we were able to distinguish between those that tested the SMK and those that tested the PCK. We used questions from the Graduate Record Examinations as a means to measure the SMK because these items were specifically designed "to indicate knowledge of the subject matter." (www.ets.org) Only geometry questions were selected from various GRE test sources. A total of nine multiple-choice GRE questions were examined (see Appendix B for examples of the mathematical questions). The results of the pre- and post-tests were compared to assess for the change in knowledge. Four categories were formed to differentiate the mathematical growth: No Growth/Decay for scores that did not change or went down; Moderate Growth for scores that grew between 1% and 15%; Significant Growth for scores that grew between 16% and 35%; and Exceptional Growth for scores that grew 36% or more.

We also selected several questions from the National Assessment of Educational Progress tests to assess each teacher's growth of PCK. The NAEP items were specifically designed to "present information on strengths and weaknesses in [secondary school] students' knowledge of mathematics and their ability to apply that knowledge in problem-solving situations." (www.nagb.org) We decided that the participants' performance on these particular questions was a sufficient tool to assess the teachers' PCK; looking at their ability to solve [secondary students oriented] problems would give insight into participants' ability to teach the concepts these problems incorporate. We selected two multiple-choice questions and two written response questions to assess the PCK (see Appendix B for examples of the instructional questions). The four categories, presented above were applied to differentiate the levels of growth: No Growth/Decay for scores that did not change or went down; Moderate Growth for scores that grew between 1% and 15%; Significant Growth for scores that grew between 16% and 24%; and Exceptional Growth for scores that grew 25% or more.

Preliminary Results

Our hypotheses: those teachers that identify strong mathematical purposes would grow mathematically; those that identify weak mathematical purposes would not grow mathematically; those that identify strong instructional purposes will grow instructionally; and those that identify weak instructional purposes would not grow instructionally. Though the preliminary results indicate that data collected from 8 teachers support the hypothesis and data collected from 10 teachers refute it, the developed framework allowed us to successfully investigate the interconnected nature of MKT. Preliminary findings suggest reflections on mathematical tasks positively affected the growth of MKT in 15 of the 18 participants; specifically, 14 participants showed an increase in PCK, 8 participants showed an increase in SMK, and 7 participants showed an increase in both areas.

Questions

What types of reflections would spark substantial growth in MKT in a mathematics content course? What are other meaningful ways of foster growth of MKT in college mathematics courses? What theoretical perspectives would provide a better lens to observe and analyze the phenomenon of developing MKT in mathematics courses through reflections?

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Appendix A:

An example of a mathematical task used in the course (Adopted from *Developing Thinking in Geometry* by Johnston-Wilder & Mason, 2005):

Task 1: Draw a triangle. Draw a square outward on each side of this triangle. Join the outer corner of each square to the nearest outer corner of the next square to form three additional 'flanking' triangles, each one between two adjacent squares. Which is the largest of the triangles? Does it depend on your starting triangle?
Task 2: Draw a triangle and label the vertices with the co-ordinates (0, 0), (1, 0), and (x, y). Draw a square outward on each side of the triangle. Join each square to its neighbor by joining the nearest vertices of each of the squares. Find the areas of all four triangles.
Task 3: Draw a triangle, draw a square outward on each side, and join the vertices of the square to get three more 'flanking' triangles. Show that each of the new triangles has the same area as the original triangle.

Reflection question: *What was the relationship between these three tasks? Could this assignment be used in your classroom? If not, why?*

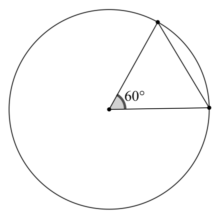
Table 1: Final rubric for the purpose used to analyze the teachers' reflections

<table>
<tr><th></th><th>0 (Undefined)</th><th>1 (Weak)</th><th>2</th><th>3 (Strong)</th></tr>
<tr><td>Mathematical</td><td rowspan="2">No purpose provided</td><td>Solution to the task or stated the mathematical concept being addressed</td><td>Provided more detail about the concept or outlined the steps required to complete the task</td><td>Explained a solution that included the connections to the context not directly used in the task</td></tr>
<tr><td>Instructional</td><td>General comment made on the strategy used to complete the task</td><td>Provided some insight on the strategy needed to teach and/or complete the task</td><td>Recognized the students learning process and how the task used pedagogical strategies and the reason for using such approach</td></tr>
</table>

Appendix B:

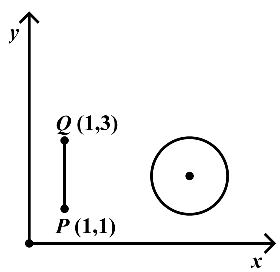
Samples of mathematical questions from the pre- and post-tests:

Select one of the following four answer choices:



a) The circumference of a circle is greater than 12
b) The circumference of a circle is less than 12
c) The circumference of a circle is equal to 12
d) The relationship cannot be determined from the information given

The figure shows line segment PQ and a circle with radius 1 and center (5, 2) in the *xy*-plane.

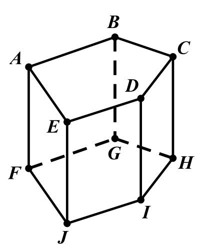


How many of the following values could be the distance between a point on line segment PQ and a point on the circle?

{2.5, 3.0, 3.5, 4.0, 4.5, 5.0, 5.5, 6.0}

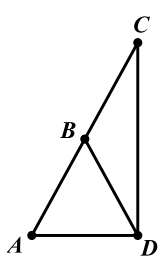
a) Only one value
b) Only two values
c) Only five values
d) All of the values
e) None of the values

Samples of instructional questions from the pre- and post-tests:



In the figure above, points A, E, and H are on a plane that intersects a right prism. What is the intersection of the plane with the right prism?

a) A line
b) A triangle
c) A quadrilateral
d) A pentagon
e) A hexagon



Write the proof in the space provided.

Given: B is the midpoint of AC and AB=BD

Prove: Angle CDA is right