A CODING SCHEME FOR ANALYZING GRAPHICAL REASONING ON SECOND SEMESTER CALCULUS TASKS

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Abstract: As a first step in studying students' spatial reasoning ability, preference, and their impact on performance in second semester calculus, I ran a pilot study to develop interview tasks and a coding scheme for analyzing the interviews. Four videotaped interviews were conducted with each of the five participants and the video was coded for graphical reasoning. I will discuss my coding scheme and share some preliminary results. I hypothesize that the coding scheme may help identify a student's preference and ability for spatial reasoning.

Keywords: Second Semester Calculus; Graphical Reasoning; Coding; Interview

Introduction and research questions

Second semester calculus is widely regarded by undergraduates as one of the most difficult or most failed math classes. Underlying the primary concepts in a second semester calculus course is the limit concept, which research has shown to be difficult for many students (e.g., Tall & Vinner, 1981). Furthermore, difficulties with the concepts of differentiation and integration are often related to deficiencies in a student's understanding of limits (Orton, 1983; Tall, 1992). Combining this with the abstract nature of the material, techniques of integration and infinite series, creates a rich environment for studying student learning.

In addition to the complexities of understanding the content, I am also interested in how students' spatial reasoning impacts their success in second semester calculus. Comacho and González-Martín (2002) investigated how students fare at performing non-routine tasks using improper integrals, as well as students' use of the graphic register versus the algebraic register in completing the tasks. In particular, several of their survey questions asked students to use graphical thinking to answer and interpret a given question. They found that students preferred to use the algebraic register, even when specifically asked to use or create a graph, and that, "generally speaking, they are unable to articulate information between these two registers" (Comacho & González-Martín, 2002, p. 9). This finding highlights the importance of the psychological perspective—considering students' spatial and symbolic abilities—when instructing them on limits. A recent study of Haciomeroglu and Chicken (2011) considered several measures of cognitive abilities and performance of high school students on three measures of mathematical performance (AP Calculus AB exam scores and scores on both the Mathematical Processing Instrument and the Mathematical Processing Instrument for Calculus). They performed a correlational analysis on their data and determined that spatial orientation ability seems unrelated to calculus, although "visualizing mathematical objects from different perspectives is crucial to understanding calculus" (Haciomeroglu & Chicken, 2011, p. 68).

The research questions my larger study seeks to answer are: (1) How is the limit concept understood by second semester calculus students across the contexts of limits of functions, definite integral approximation and error, improper integrals, and infinite sequences and series? (2) Do individual differences in students' visual and symbolic reasoning skills impact students' ability to understand the limit in calculus or their performance on tasks? The purpose of this preliminary study was to develop tasks and a schema for analyzing the interviews. These research questions will hopefully be addressed in the larger follow-up study currently underway.

As an initial step to answering these research questions, I ran a pilot study during Spring 2012 that I will discuss here. I had two primary goals for this study: (1) develop and test tasks for interviews and (2) develop a coding scheme for analyzing the interview videos. I hope to get feedback on the research goals as well as the coding scheme.

Methods/subjects

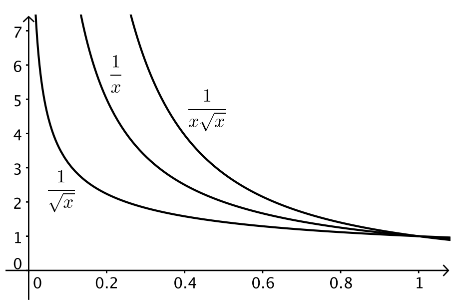
The five participants in the study were chosen from one instructor's sections of a second semester calculus course at a small Midwestern University. I will refer to them by the pseudonyms Daniel, Jon, Laura, Sarah, and Travis. The participants were chosen in part because they had no previous second semester calculus experience, including no AP Calculus BC experience. The participants were all freshman, with an average age of 19 years old, with STEM majors. They volunteered in exchange for extra credit and met with the researcher for approximately one hour per week for a total of six weeks. I will focus on four weeks of videotaped interviews here (a total of 20 interviews). In each of these interviews, students worked individually on specific mathematical tasks: see Figure 1 for examples. Students were instructed to "think aloud" while solving the tasks. If needed, students were asked to clarify their thinking.

1. Find $\lim_{x\to 1} \frac{x^2-1}{x-1}$.
2. Find $\lim_{x\to\infty} \frac{x^5}{2^x}$.
3. Do the following integrals converge or diverge?

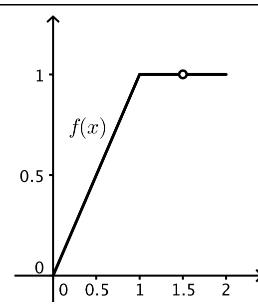
a. $\int_0^1 \frac{1}{\sqrt{x}}dx$

b. $\int_0^1 \frac{1}{x\sqrt{x}}dx$

c. $\int_0^1 \frac{1}{x}dx$



4. Find $\int_0^2 f(x)dx$.



5. Does $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\ldots$ converge or diverge? Can you draw me a picture of what that series represents? Is there any connection to $\int_1^\infty \frac{1}{x}dx$?

(Note: we do not teach students the harmonic series in second semester calculus)

Figure 1: Examples of tasks

The resultant video from the interviews was coded for use of graphics when solving tasks. Graphs arose in four primary ways: a graph was given as part of the task; the participant created a graph with prompting; the participant created a graph without prompting; or the participant reused a previously created graph. See Figure 2 below for the frequency of each type. Note that the number of graphs given as part of the task is not consistent: depending on how previous participants fared on an individual problem, the presentation of the problem was sometimes adjusted. In addition, some participants frequently created graphs without prompting, while others required prompting. In each instance where a graph occurred, the graph was labeled in three separate categories, as appropriate: graph creation, reasoning from graph, and connection to symbolic reasoning (see Figure 3 below). Frequently more than one label from a category would be applied to an instance. For example, when using a graph, a participant would initially reason incorrectly with the graph. After consideration, correct and helpful reasoning would occur. Such an instance would be labeled for both "Incorrect reasoning with graph" and "Correct and helpful reasoning from graph."

In addition, I am particularly interested in those instances where a participant solves a problem symbolically, solves the same problem graphically, gets two different answers, and then tries to resolve the two answers. Several tasks were developed to generate this conflict. For example, Daniel solved the problem $\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$ and was able to correctly do the symbolic manipulation to get the limit of 2. Then, after prompting, Daniel created an incorrect graph (see Figure 4 below). In using this incorrect graph, Daniel had both consistent and inconsistent reasoning: the limit does not exist because it is different on either side; the limit is x because it is a slant asymptote. In either case, his reasoning contradicted his symbolic reasoning and he was unable to resolve this conflict. This instance, where the graph was created with prompting, was labeled as "Student created incorrect graph," "Consistent reasoning with incorrect graph," "Inconsistent reasoning with incorrect graph," and "Reasoning contradicts symbolic reasoning."

	Daniel	Jon	Laura	Sarah	Travis
Graph given as part of task	14	18	16	15	15
Graph created with prompting	7	9	15	5	14
Graph created without prompting	7	5	4	14	5
Reused previously created graph	3	6	4	4	3

Figure 2: Types of occurrences and preliminary results

	Daniel	Jon	Laura	Sarah	Travis
Graph creation:					
Easily produced correct graph	9	11	10	3	15
Able to produce correct graph after prompting or errors	0	3	6	4	5
Unable to create graph after help	1	0	1	0	0
Interviewer generated graph after unsuccessful attempt	0	0	1	0	0
Created incorrect graph and used it to complete task	4	0	0	9	0
Connection to symbolic reasoning:					
Reasoning contradicts symbolic reasoning	4	1	4	6	3
Reasoning supports symbolic reasoning	7	5	9	4	5
Reasoning from graph:					
Correct and helpful reasoning from graph	12	24	21	13	20
Correct but unhelpful reasoning from graph	3	2	5	1	2

Incorrect reasoning with graph	12	15	11	15	16
Did not use graph for reasoning	3	0	1	2	1
Unable to reason with graph	2	1	0	4	3
Consistent reasoning with incorrect graph	4	0	0	8	0
Inconsistent reasoning with incorrect graph	1	0	0	1	0
Total graphs considered	*31*	*38*	*39*	*38*	*37*

Figure 3: Labels for each occurrence and preliminary results

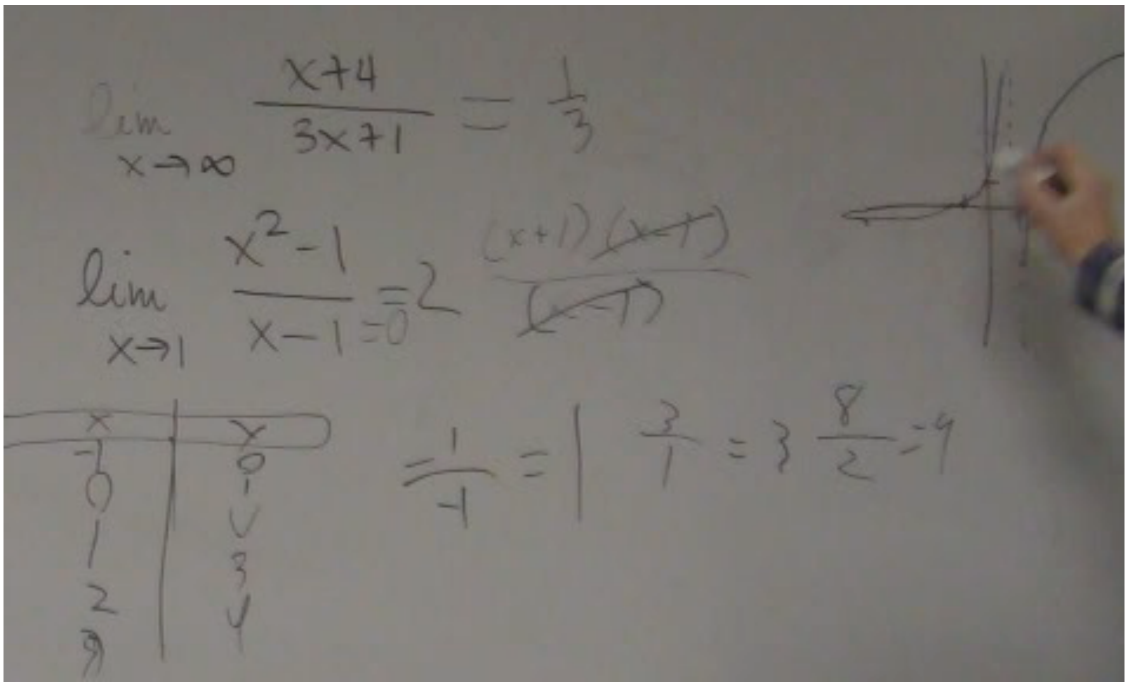


Figure 4: Daniel's solution to a limit problem

Results and future directions

From the preliminary data in Figure 3 above, we can hypothesize that Daniel and Sarah have less graphical reasoning ability than the other participants because they created more incorrect graphs (4 and 9, respectively) and fewer correct graphs (9 and 3, respectively) than the other participants. In addition, both Daniel and Sarah have less "Correct and helpful reasoning from graph" than other participants (12 and 13, respectively), supporting the hypothesis that they have less graphical reasoning ability. Sarah's inability to create correct graphs contrasts her strong willingness to create graphs without prompting: Sarah created 14 graphs without prompting. From this, I hypothesize that although Sarah may have less graphical reasoning ability, Sarah may prefer to use graphical reasoning when solving calculus problems. Finally, Daniel used fewer graphs than the other participants, which may indicate Daniel's preference for using symbolic reasoning when solving calculus problems.

During 2012-2013, I will be running a study using the piloted tasks as well as several psychological measures of spatial ability and preference, including Haciomeroglu and Chicken's (2011) Mathematical Processing Instrument for Calculus, a shortened version of Suwarsono's Mathematical Processing Instrument (1982), as well as the Form Board Test, Paper Folding Test, Card Rotations, Cube Comparisons, Diagramming Relationships and Nonsense Syllogisms Test from the Kit of Factor-Referenced Cognitive Tests (Ekstrom et al, 1976). I hope to be able to connect the interview data to the measures of spatial ability and preference. In addition, I will be collecting participants' midterm and final exams and thus will be able to track their performance in their second semester calculus course.

Questions for the audience

1. Would it be better to keep track of each occurrence of correct or incorrect reasoning with a graph, instead of just labeling each use of a graph as having correct or incorrect reasoning? Often a student would have multiple occurrences of correct or incorrect graphical reasoning within a task and in the current scheme, these would only be counted once per graph.
2. What other information might be useful to track from these interviews?
3. What other changes or additions should I make in the next round of coding?

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