PRE-SERVICE TEACHERS' MATHEMATICAL KNOWLEDGE FOR TEACHING AND CONCEPTIONS OF TEACHING EFFECTIVENESS: ARE THEY RELATED?

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Mathematical knowledge for teaching (MKT) is essential for effective teaching of elementary mathematics. Given the importance of MKT, MKT and conceptions of teaching effectiveness should not develop independently. The purpose of this study was to examine whether and how K-8 pre-service teachers' MKT and personal mathematics teacher efficacy beliefs are related. Results indicated overconfidence in teaching ability was prevalent, with the majority of participants exhibiting a strong sense of personal mathematics teacher efficacy but low levels of MKT. Pre-service teachers with high levels of MKT, however, reported a more accurate assessment of their teaching effectiveness. Results also indicated that examining pre-service teachers' self-evaluations of MKT is helpful for understanding pre-service teachers' personal mathematics teacher efficacy beliefs. Moreover, the results of this study point to the inadequacies of existing measures of teacher efficacy beliefs that do not parse out differences in efficacy beliefs according to a number of contextual factors.

Key words: Mathematical Knowledge for Teaching, Pre-service Teacher Education, Efficacy Beliefs

Introduction and Theoretical Background

Personal mathematics teacher efficacy beliefs (PMTE beliefs) are a teacher's beliefs about her abilities to teach mathematics effectively (see, e.g., Tschannen-Moran & Hoy, 2001). How effective a teacher will be for student learning depends on the level of mathematical knowledge that the teacher has. In particular, previous research has indicated that teachers with higher levels of *mathematical knowledge for teaching* (MKT) are more effective teachers (Ball, Thames, & Phelps, 2008; Hill, Rowan, & Ball 2005). Therefore, if a teacher is to develop accurate views of her teaching effectiveness, these views should be connected to the level of MKT that the teacher has.

Potential relationships between teacher efficacy beliefs and content knowledge for teaching have been examined in previous research. Such studies have produced inconsistent results, with efficacy beliefs and content knowledge for teaching found to be positively correlated (Swars, Smith, Smith, & Hart, 2009), weakly positively correlated (McCoy, 2011), negatively correlated (Wenner, 1993), or uncorrelated (Swars, Hart, Smith, & Tolar, 2007; Wenner, 1995).

These inconsistent results might be due to a mismatch between assessments of mathematical knowledge for teaching and assessments of personal mathematics teacher efficacy beliefs. Measures of mathematical knowledge for teaching are designed to reflect what teachers actually do in mathematics classrooms, but measures of personal mathematics teacher efficacy beliefs are typically not situated in classroom tasks. Additionally, typical efficacy-beliefs measures do not contain any actual mathematics. For example, both Swars et al. (2009) and McCoy (2011) measured personal mathematics teacher efficacy beliefs with a subscale of the Mathematics Teaching Efficacy Beliefs (MTEBI) Instrument. MTEBI items do not situate teacher efficacy beliefs in any classroom situations and do not contain any mathematics. Rather, items refer to mathematics teaching in a general way, such as in the items "I know how to teach mathematics concepts effectively" and "I wonder if I have the skills necessary to teach mathematics" (Enochs, Smith, & Huinker, 2000, p. 200-201).

Thus, measures of mathematical knowledge for teaching and measures of personal mathematics teacher efficacy beliefs are apparently disconnected. Measures of mathematical knowledge for teaching assess mathematical knowledge for teaching specific mathematical content, but measures of personal mathematics teacher efficacy beliefs measure beliefs not tied to specific content. Efficacy beliefs are task-specific constructs (Bandura, 1986), so personal mathematics teacher efficacy beliefs are likely to vary based on the content to be taught. One alternative is to measure both personal mathematics teacher efficacy beliefs and mathematical knowledge for teaching in the context of specific mathematical teaching tasks. This is the approach taken in the current study.

The overall purpose of this study was to examine whether and how pre-service-teachers' personal teacher efficacy beliefs were related to their mathematical knowledge for teaching. No empirical studies identify whether there is an ideal relationship between these two constructs. However, given the importance of mathematical knowledge for teaching, having personal mathematics teacher efficacy beliefs and mathematical knowledge for teaching that are aligned is perhaps preferable. In this article, personal mathematics teacher efficacy and mathematical knowledge for teaching will be considered *aligned* when higher levels of personal mathematics teacher efficacy accompany higher levels of mathematical knowledge for teaching, or lower levels of personal mathematics teacher efficacy accompany lower levels of mathematical knowledge for teaching. Pre-service teachers who have personal mathematics teacher efficacy beliefs and mathematical knowledge for teaching that are misaligned might overestimate their teaching abilities. Such overestimation is a potential barrier to improving one's teaching, as dissatisfaction with one's performance can be a catalyst for change (Guest, Regehr, & Tiberius, 2001; Wheatley, 2002).

The study aimed to address the following research questions: (1) How prevalent is alignment of personal mathematics teacher efficacy beliefs and mathematical knowledge for teaching? (2) What differences are evident between pre-service teachers with low mathematical knowledge for teaching and those with high mathematical knowledge for teaching with respect to alignment of personal mathematics teacher efficacy beliefs and mathematical knowledge for teaching? (3) How do pre-service teachers' self-evaluations of their mathematical knowledge for teaching relate to their actual mathematical knowledge for the aligned and misaligned groups?

Methods

Forty-two K-8 pre-service teachers participated in the study. The study was conducted at a medium-sized university in the Northeastern United States. All participants were enrolled in the second course of a three-course series of mathematics courses required for the teacher education program at this university. Participants were randomly selected for participation from the pool of 209 students enrolled in this second course.

Pre-service teachers first participated in a 90-minute semi-structured interview in which they were asked to respond to four *Teaching Scenario Tasks*. For each task, pre-service teachers first gave a written response and then were asked to explain their answers orally. All interviews were audio-recorded; recordings were used to supplement written responses. Two to four weeks after the semi-structured interview, pre-service teachers participated in an individual 60-minute session to complete four *MKT tasks*.

Each of the Teaching Scenario Tasks presented a scenario that required the pre-service teachers to give a conceptual explanation to a student's "why" question about a problem involving fractions. A sample Teaching Scenario Task is displayed in Figure 1. Pre-service teachers' responses to the prompt "I am confident that my explanation would be effective in helping the students understand the relevant concepts" measured personal mathematics teacher efficacy for the given task. Responses of strongly disagree or disagree were considered *low PMTE* and responses of strongly agree or agree were considered *high PMTE*. Each of the Teaching Scenario Tasks had the same format.

Each MKT Task was designed to measure participants' MKT for the mathematics involved in the corresponding Teaching Scenario Task. Figure 2 contains the MKT Task that corresponds to the Teaching Scenario Task in Figure 1. For each task, a list of subcomponents involved in giving a complete mathematical explanation was constructed. Pre-service teachers could obtain a score of 0, 1, or 2 for each subcomponent. The total MKT score for a particular task was the sum of these scores across all subcomponents. Inter-rater reliability scores were obtained for MKT-coding on each task. The ratings for Tasks 1, 2, 3 and 4 were 81%, 82%, 82%, and 92% respectively.

For each task, a participant was considered *high* with respect to her exhibited MKT if her exhibited MKT score was at least 70% of the total possible score. Pre-service teachers with scores less than 70% of the total possible score were considered *low* with respect to her exhibited MKT. A cut-off of 70% was used because of the high standard for each subcomponent; that is, obtaining a score of 2 was difficult.

Results

On each MKT Task, pre-service teachers were asked to respond to the prompt "I am confident that I understand the mathematical concepts in this task." Responses to this prompt are displayed in Table 1. One notices from the table that pre-service teachers rated Task 2 most understandable mathematically and Task 4 least understandable mathematically. This result will be helpful in understanding the overall results for each of the three research questions.

To address research question 1, percentages of pre-service teachers for whom the two constructs were aligned or misaligned were calculated, as shown in Table 2. Misalignment was evident on Tasks 1 and 3 with a majority of pre-service teachers exhibiting high PMTE beliefs but low MKT on these tasks. On Tasks 2 and 4, the tasks rated most and least understandable respectively, higher frequencies of pre-service teachers had aligned PMTE beliefs and MKT. Overall, misalignment was prevalent, with 90 of the total 168 cases (42 participants on each of 4 tasks) falling into the High PMTE beliefs/Low MKT category.

To address research question 2, percentages of pre-service teachers with aligned PMTE beliefs and exhibited MKT by task and by level of MKT for that task were calculated, as shown in Table 3. One notices that high-MKT pre-service teachers tended to fall into the aligned category with greater frequency than low-MKT pre-service teachers, except on Task 4. Task 4 was, again, the task rated least understandable by the sample of pre-service teachers as a whole.

To address research question 3, the self-ratings of mathematical understanding that pre-service teachers gave on a particular task were compared to pre-service teachers' exhibited MKT for that task, as shown in Table 4. For the misaligned groups on each task, self-ratings of MKT and exhibited MKT were uncorrelated. On Tasks 2 and 4, pre-service teachers in the aligned groups who tended to rate their MKT higher also tended to exhibit higher levels of MKT.

Implications

The findings of this study indicate that pre-service teachers' personal mathematics teacher efficacy beliefs are more nuanced than previous research has suggested. The fact that results from the four tasks did not look identical suggests that personal mathematics teacher efficacy beliefs are highly contextual and, as such, should be measured in varying contexts. In particular, pre-service teachers' self-evaluations of their mathematical knowledge for teaching, with respect to how understandable the mathematics in a task was, seemed to influence pre-service teachers' personal mathematics teacher efficacy beliefs. Moreover, personal mathematics teacher efficacy beliefs were better aligned with pre-service teachers' evaluations of their mathematical knowledge than with pre-service teachers' actual mathematical knowledge.

Misalignment of personal mathematics teacher efficacy beliefs and mathematical knowledge for teaching was, overall, prevalent. In particular, the overall frequency of the “High PMTE beliefs/Low MKT” category (54%), arguably the most problematic category, is noteworthy. Many pre-service teachers likely need help in assessing their teaching effectiveness accurately, help that teacher educators might need to provide during teacher education programs. Pre-service teachers in the “High PMTE beliefs/Low MKT” category are likely those whose sense of personal mathematics teacher efficacy is inaccurate and whose MKT needs development. This problem is compounded by the fact that such pre-service teachers are less likely to recognize that they have low MKT (e.g., see Kruger and Dunning, 1999).

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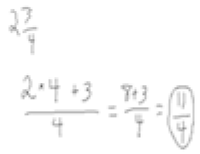
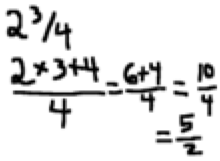
Two students in your fifth-grade class, Joe and Amy, are trying to convert a mixed number 2 ¾ to an improper fraction. Each of their solutions is shown below:

Joe



Amy



1. Joe and Amy both say that they are using a rule that they know for rewriting mixed numbers as improper fractions. They are not sure which rule gives the right answer. If you were Joe and Amy's teacher, how would you explain this to your class so that they would understand?

2. Thinking of yourself as Joe and Amy's teacher, respond to the following:

I am confident that my explanation would be effective in helping the students understand the relevant concepts.

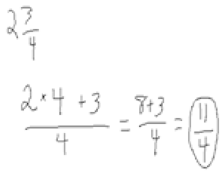
Strongly Disagree Disagree Agree Strongly Agree

(circle one)

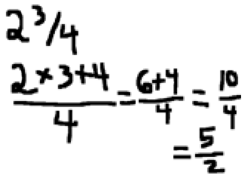
Why did you choose this rating?

Figure 1. Sample Teaching Scenario Task.

Joe



Amy



1. Show with a picture which student is correct. Use the picture to show why this procedure works. Give a detailed conceptual explanation that explains your reasoning for each step of the procedure and use your picture to explain your reasoning.

2. Please respond to the following:

I am confident that I understand the mathematical concepts in this task.

Strongly Disagree Disagree Agree Strongly Agree

(circle one)

Figure 2. Sample Participants' MKT Task.

Table 1
Percentages of pre-service teachers' MKT self-evaluation ratings by task (n = 42)

Task	MKT Self-evaluation rating: "I am confident that I understand the mathematical concepts in this task"			
	Strongly Disagree	Disagree	Agree	Strongly Agree
Task 1	0	14	38	48
Task 2	0	0	43	57
Task 3	0	2	57	40
Task 4	2	17	62	19

Table 2
Percentages of pre-service teachers with aligned or misaligned PMTE beliefs and total MKT score by task (n = 42)

Task	Relationship between PMTE beliefs / Total MKT Score			
	High/High	High/Low	Low/High	Low/Low
Task 1	21	71	0	7
Task 2	43	36	5	17
Task 3	21	64	0	14
Task 4	14	43	10	33

Table 3
Percentages of pre-service teachers with aligned PMTE beliefs and total MKT score by task and by level of MKT shown on the task (n = 42)

Task	Percentage of High MKT and Low MKT pre-service teachers with aligned PMTE beliefs and total MKT score	
	High MKT	Low MKT
Task 1	100	9
Task 2	90	32
Task 3	100	18
Task 4	60	44

Table 4

Results from Spearman's rho tests for examining the relationship between MKT self-evaluation and total MKT score

	Aligned Group			Misaligned Group			Entire Group		
Task	n	value of ρ	p-value	n	value of ρ	p-value	n	value of ρ	p-value
Task 1	12	.331	.293	30	.226	.229	42	.335*	.030
Task 2	25	.437*	.029	17	.120	.645	42	.340*	.028
Task 3	15	.283	.304	27	-.043	.835	42	.128	.423
Task 4	20	.616**	.004	22	.052	.820	42	.334*	.031

*$p \leq 0.05$; ** $p \leq 0.01$