TEACHING UNDERGRADUATE CALCULUS FOR TRANSFER: A QUALITATIVE CASE STUDY OF THE CALCULUS SEQUENCE AT ONE LIBERAL ARTS COLLEGE

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At small liberal arts colleges, a single calculus sequence must successfully accommodate students from various majors, such as mathematics, biology, chemistry, and economics. This qualitative case study considers mathematics professors' perspectives about the required nature of calculus in various disciplines, attempts to identify how calculus instructors teach with the aim of preparing students to apply calculus knowledge in their future coursework, and how the disciplinary focus of their students affects professors' design and teaching of calculus courses. Framed using aspects of teaching and learning shown to promote transfer of knowledge, results suggest that the professors teach for understanding and allow in-class processing time, but could improve their emphasis on applying calculus in non-mathematics disciplines. This study contributes to the growing body of undergraduate mathematics education research intended to document undergraduate teaching practices.

Keywords: Undergraduate Calculus, Transfer, Qualitative Case Study

Objectives

Undergraduate courses like the calculus sequence serve students from various majors, such as mathematics, biology, chemistry, and economics. Small liberal arts colleges (SLACs) do not have the ability to offer multiple calculus courses specifically for students from these departments, but instead must offer one calculus sequence to successfully serve these students and departments. Newly added majors in Biochemistry and Molecular Biology at the SLAC in this study, both of which require significant calculus, are changing the composition of students taking the calculus sequence to include more science students. This change in student composition prompts the need to examine how calculus instructors design and teach courses in the calculus sequence, largely populated by non-mathematics students. Specifically, research is needed to determine how calculus instructors prepare students who are required to take calculus for their major to use calculus in their major disciplines.

This qualitative case study considers mathematics professors' perspectives about the required nature of calculus in various disciplines, and attempts to identify aspects of professors' design and teaching of calculus courses that might increase students' ability to use their calculus knowledge in subsequent courses. In particular, the study asks: (1) Why do mathematics professors believe specific departments on campus require their students to take all or part of the calculus sequence?; and (2) How do calculus instructors use teaching methods known to improve learning for transfer?

Background Literature and Theoretical Framing

While it is often acknowledged that calculus serves a wide audience (Bressoud, 1992), little research has formalized how calculus instruction might be designed to accommodate the different disciplines that make up a typical calculus course at a SLAC. A personal account of a successful restructuring of Calculus I that was recently undertaken at Macalester College, a small liberal arts institution, illustrates how knowledge about calculus use in non-mathematics departments can be used to inform calculus course design (Bressoud, 2008). There have also been calls for quantitative skills in disciplines such as biology (e.g., Bialek & Botstein, 2004) and surveys attempting to connect quantitative literacy with economic literacy (e.g., Schuhmann, McGoldrick, & Burrus, 2005). Despite these calls, surveys, and personal accounts of relevant calculus restructuring, research has not been conducted to formalize how, or if, calculus professors attempt to teach calculus for transfer of knowledge to non-mathematics disciplines. In fact, empirical research in undergraduate mathematics education is limited, and calls have been made for more research to illuminate college teaching practices (Speer, Smith, & Horvath, 2010).

In order to characterize how mathematics instructors design and teach calculus courses for transfer of knowledge into other disciplines, both knowledge about professors' perceptions of the required nature of calculus in other disciplines and knowledge of the ways in which professors currently teach calculus courses are needed. According to cognitive learning theory, learning can be considered as knowledge construction (Mayer, 1992). In this perspective, learners control their own cognitive processes during learning, and teachers are also participants in the learning process, actively constructing meaning during learning situations. As a result, finding ways to help students effectively process information becomes a central instructional issue. This study explored how calculus instructors teach calculus for transfer, which Bransford, Brown, and Cocking (2000) define as “the ability to extend what has been learned in one context to new contexts” (p. 51).

Initial learning is essential to transfer, and many types of learning experiences have been shown to promote transfer, such as learning with understanding as opposed to only memorizing facts or procedures without a connection to why those procedures or facts are used or work, learning with time to process complex material, and learning with frequent feedback focused on when, where, and how to use knowledge (Bransford et al., 2000). Bransford et al. also point out that research has shown that students' knowledge transfer is increased when potential ways in which the knowledge they are learning might be useful in the future are highlighted during initial learning. Beyond the types of initial learning experiences that have been shown to promote transfer, learning in multiple contexts can promote transfer of knowledge since students are able to create flexible knowledge of the topic by abstracting general features (Bransford et al., 2000).

Readily explorable in calculus teaching practices through interviews with professors, classroom observations, and examination of course materials, five characteristics of teaching and learning for transfer supported by the empirical literature framed this study: learning with understanding, giving processing time, giving frequent feedback focused on understanding, noting transfer implications during original study, and learning in multiple contexts. Although the professors were not familiar with the concept of knowledge transfer from a formal educational learning theory perspective (and I did not present these aspects of teaching as “ways to promote transfer” during interviews), they were aware that certain of these ideas, such as connecting concepts to future applications, could be of benefit to student learning.

Research Methodology

This study took the form of a qualitative case study because it provides an “in-depth description and analysis of a bounded system" (Merriam, 2009, p. 40), the calculus sequence at one liberal arts college. Data were collected in one Calculus I course and two Calculus II courses that were conducted during the Spring 2012 semester. Mathematics professors were the key informants in this study because they represent experts in calculus. Their perceptions of calculus, both as key in mathematics and as required for other disciplines, forms the basis of how the calculus sequence is designed and executed. Focal mathematics professors represented the mathematics department's general views as conveyed in an initial survey, and can therefore been seen as a typical (Patton, 2003) and purposeful sample (Merriam, 2009; Patton, 2003). Selecting focal professors who self-identified as having information to give about calculus is one way I ensured "information-rich" subjects to study (Merriam, 2009).

The data collection proceeded in four phases. An initial email survey provided basic information about departmental beliefs about calculus teaching and learning and was used to select the three focal calculus professors. Preliminary individual semi-structured interviews asked professors to provide perceptions of why calculus is required in other disciplines, the knowledge or skills they believe their calculus students obtain, and to discuss how (if at all) they believe they teach calculus so that their students are prepared to use calculus in their major disciplines. The third phase of data collection was a 50 minute class session observation, during which I was an "observer participant" (Merriam, 2009) generating knowledge of how calculus is used in the classroom for triangulation with the email survey results and interviews. Classroom observations informed the final phase of data collection, individual follow-up semi-structured interviews, where professors were asked to describe their calculus courses in more detail based on observation of their class and examination of their course materials.

Data, consisting of transcripts from interviews, field notes from observations, and course materials, were analyzed for the ways in which the calculus instructors teach for transfer, focusing on the five characteristics of learning and teaching that promote transfer. Collecting data from several different sources is one way I attempted to ensure internal validity, as it provided opportunities for triangulation of data (Merriam, 2009). My analysis began with an initial reading of the entire data set (Emerson, 1995), during which I asked questions of the data set (Emerson, 1995) (such as, "Where and how does the instructor emphasize understanding? And, where and how does the instructor emphasize computational procedures?"), as I initially open coded the data. Themes and patterns in the data were used to develop an analytic code list based on the five aspects of teaching that promote transfer, and focused coding of the data was undertaken (Emerson, 1995). In order to make assertions about answers to the research questions, this coded data was used to develop themes, make comparisons, and determine how the particulars of the data could be generalized (Miles & Huberman, 1994). Themes that appeared were often investigated further, and claims were generated. Then, data was searched for both confirming and disconfirming evidence of these claims (Miles & Huberman, 1994). Finally, these themes and general patterns, drawn from the interviews, observations, and course documents, were used to create hypotheses about how calculus professors teach for transfer, a few of which will now be described.

Results

When asked if they knew why other departments required calculus, all three calculus professors could quickly state applications of calculus in non-mathematics disciplines which require calculus of their students. All of the calculus professors in this study also articulated a justification for non-mathematics departments requiring calculus by drawing on their personal knowledge of these various disciplines, such as economics, chemistry, biology, and physics. One professor's explanation of this is typical of all three professors: “So, there's kind of the obvious idea that calculus tools are very good at describing and modeling change. So, any time things are changing, calculus may be useful.” The other two professors answered in a similar way, by giving examples from chemistry, biology, and physics, that were admittedly from their “personal contact” with others studying applied mathematics or simply from their personal knowledge of the discipline. Professors' conceptions of calculus, and ideas about what students should be learning in calculus, appear to come primarily from their own knowledge of calculus and personal experiences with non-mathematics disciplines.

Instructors of calculus at this SLAC are already teaching in some ways that might prepare students to transfer their calculus knowledge to future coursework. One such result will be highlighted here. During interviews, when outlining the knowledge and skills that they believe students should leave calculus courses knowing, each of the calculus professors mentioned both computational skills (e.g., being able to calculate the derivative of a polynomial) and some type of conceptual understanding, often described as deeper understanding, intuition, or fluency (e.g., knowing that the derivative is a rate of change, when/why that information might be useful). For example, one professor describes computational skill as follows: “...the mechanical skills are the symbol pushing. The, if I give you something, can you manipulate the symbols to get what I'm asking for?” While conceptual understanding gets at something deeper: “When I say intuition I mean a better sense about what is going on...it's trying to give them a framework, so that when they see something that can be expressed as a rate of change, that they understand that they can manipulate that to get more information.”

Each calculus professor articulated the desire for students to have conceptual understanding, noting something similar to, “I'm not particularly interested, as a college teacher, in training them to follow a memorized pattern. I want them to think about what's going on.” The professors also made attempts to foster such understanding during observed class sessions, by asking students to always justify their reasoning, suggesting that calculus students need more than computational skills to learn calculus successfully. Emphasis on conceptual understanding was particularly noted when professors gave students time to actively process the material being covered in class.

Despite the fact that conceptual understanding appears to be the ultimate goal for these calculus professors, each professor also acknowledged the foundational importance of computational skill. In fact, mechanical skill plays a large role in these calculus courses, and conceptual understanding may be just emerging for calculus students. Importantly, professors expressed difficulty in being able to assess whether students possess this conceptual understanding without having one-on-one conversations with students. One professor said, “I think that I give tests because I have to give grades, I don't think tests measure what people know. … So, I find out the most about what students know when they come in and sit down at that table right there and we talk about things that they are wrestling with.” It would be of benefit to further study how assessments might be adjusted to match what professors hope their students are taking away from calculus courses.

Areas of calculus teaching that could be improved to encourage promotion of transfer were also articulated during the study. Considering one example, calculus professors admit in interviews that they do not specifically focus on bringing in applications of calculus to non-mathematics disciplines, although course observations reveal that examples are widely used in these courses to illustrate calculus concepts and professors admit that applications could benefit student learning. One professor articulated this by saying, “I make small effort to bring in examples from more than just the areas that are in the textbook. I do know that seeing something applied in a discipline that they are actually interested in makes a difference for their understanding.” Additionally, one professor articulated how these examples are chosen so that students are able to understand both the specific variety of applications of a particular concept and the general abstract ideas underlying those concepts. Calculus textbooks include examples from other disciplines, and professors do make use of those built in applications at times, but do not go out of their way to find disciplinary applications outside of mathematics.

The professors articulated two reasons for not including more applications. Balancing the interests of students from a variety of majors: “So, what might be great for 5 of your 20 students, is worthless for the others.” And, a notable desire to maintain some type of a “traditional” calculus sequence: “I think, what are the primary things I want them to understand from the perspective of what mathematics *is*? And then, I certainly use examples to illustrate the utility of those concepts. But, to me, they are just examples.” Emphasizing calculus applications in other disciplines may prove to be a challenge worth further study in these foundational calculus courses.

Interview transcripts and field notes from classroom observations provide ample data to explore how the professors in this mathematics department currently teach calculus for transfer and how teaching might be altered to further promote such transfer, this is a small sample of the major findings.

Implications and Significance

Several opportunities for future research based on this study have already been noted. In particular, it could be beneficial to study calculus assessments and potential ways of improving assessment of students' conceptual calculus knowledge and to further explore whether calculus instructors might be able to increase student understanding by making more connections to disciplines outside of mathematics. This research also has direct benefits for the SLAC in this study as it can serve as the foundation for justifying restructure of the calculus sequence or emphasizing particular teaching practices. A limitation of this study is that it only focuses on mathematics professors' views of calculus teaching at the university in question. Justification for a restructuring of the calculus sequence to meet student needs could greatly benefit from both perspectives of professors in non-mathematics disciplines requiring calculus of their majors and student perspectives of the calculus sequence in relation to their studies.

SLACs also serving a variety of students with a single calculus sequence may benefit from gaining knowledge of some existing teaching practices that might promote the transfer of calculus knowledge. While personal accounts of teaching experiences exist, empirical research in undergraduate mathematics education is limited, and calls have been made for more research to illuminate college teaching practices (Speer et al., 2010). Making clear how calculus instructors design and teach calculus to ensure that students are acquiring the knowledge they need for success in their chosen majors will contribute to the growing field of undergraduate mathematics education research documenting undergraduate teaching practices.

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