Building knowledge for teaching rates of change: Three cases of physics graduate students

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Over the past two decades education researchers have demonstrated that various types of knowledge, including pedagogical content knowledge, influence teachers' instructional practices and their students' learning opportunities. Findings suggest that by engaging in the work of teaching, teachers acquire knowledge of how students think, but we have not yet captured this learning as it occurs. We examined whether novice instructors can develop such knowledge via the activities of attending to student work and we identified mechanisms by which such knowledge development occurs. Data come from interviews with physics graduate teaching assistants as they examined and discussed students' written work on problems involving rates of change. During those discussions, some instructors appear to develop new knowledge–either about students' thinking or about the content—and others did not. We compare and contrast three cases representing a range of outcomes and identify factors that enabled some instructors to build new knowledge.

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From research on teachers, it appears that particular types of knowledge used in teaching correlate with reform-oriented teaching practices and with student achievement (Ball, Hoover Thames, & Phelps, 2008; Fennema et al., 1996; Hill, Ball, & Schilling, 2008). Knowledge of student thinking, a subset of pedagogical content knowledge (PCK) (Shulman, 1986), has been found to play prominent roles in teachers' practices. Although sometimes learned in teacher preparation or professional development, findings suggest that it is often via the work of teaching that teachers have opportunities to acquire knowledge of how students think (see, e.g., Franke, Carpenter, Levi, & Fennema, 2001). To date, however, researchers have not captured this learning as it occurs.

We sought to document the use and genesis of some knowledge for teaching through task-based interviews with physics graduate students as they examined student solutions to physics problems. Although drawn from research on student thinking about physics, the task is highly mathematical in nature and is, at its core, about rates of change. Our findings indicate that, during these discussions, some graduate students appeared to develop new knowledge–either about student thinking or about the content–and others did not. We compare and contrast three cases that represent a range of outcomes. Our analysis sheds light on mechanisms that enabled two of the instructors to build new knowledge.

Theoretical Perspective

We investigate issues of knowledge development from a cognitive theoretical perspective because of the prevalence of the cognitive perspective in the research on knowledge and knowledge development as well as the primarily individual nature of the out-of-classroom teaching work that is the focus of our investigation. This perspective has been used productively to examine teachers' knowledge and its roles in teaching practices (Borko & Putnam, 1996; Calderhead, 1991, 1996; Escudero & Sanchez, 2007; Schoenfeld, 2000; Sherin, 2002). In such an approach, knowledge is seen as a key factor influencing teachers' goals and the ways they work to accomplish those goals as they plan for, reflect on, and enact instruction.

Knowledge That Shapes Teaching Practices

As noted above, research indicates that particular types of knowledge are linked with reform-oriented teaching practices and student achievement. Knowledge of student thinking, a subset of PCK plays prominent roles in this genre of research. Findings have informed the design of professional development materials and programs for elementary and secondary school teachers. This is a very positive development in the education community that may have important impacts on the teaching and learning of school mathematics. However, teachers at the college-level typically have little or no pre-service preparation (Shannon, Twale, & Moore, 1998) and opportunities for in-service professional development focused on teaching are scarce. As a result, much of the development that turns a novice college teacher into a knowledgeable, experienced teacher occurs "on the job" and the need to understand how learning occurs for this population of teachers while doing the work of teaching is especially important.

What the community lacks is insight into how and why that on-the-job learning occurs as well as the particular conditions and contexts needed for that learning to occur. Those insights could inform the design of professional development so teachers have the best opportunities possible to acquire knowledge relevant for teaching. To assist the community with these issues, we pursued these research questions: Can graduate student instructors develop knowledge while doing the work of teaching? If so, how does that knowledge development occur?

Methods

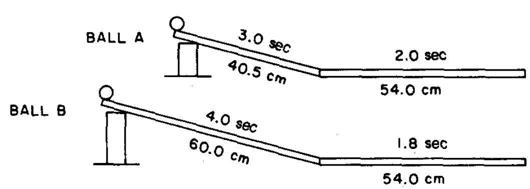
One-on-one task-based interviews were conducted with seven physics graduate student instructors. During the interviews, participants considered several kinematics problems drawn from literature on student difficulties with the concepts of velocity and acceleration (Beichner, 1994; Trowbridge & McDermott, 1981). For each problem, participants were asked to (i) solve it and give reasoning, (ii) discuss what one would need to know to solve it, (iii) generate and discuss possible student approaches, and (iv) examine and discuss samples of student work.

From the seven audio-recorded interviews, we selected three for cross-case analysis (Yin, 1989). One interview was selected for closer analysis based on its *inherent interest* to our research questions. During this interview, Jamie developed an increasingly specific and accurate diagnosis of a known common student difficulty. The other cases were selected for their *contrastive value*. Sam developed better understanding of the underlying physics by attending to student work and Alex struggled to understand the problem or student thinking.

To conduct the analysis, we drew on research findings about student thinking about these problems (Trowbridge & McDermott, 1981) and methods for fine-grained analysis of teacher thinking and practices (Schoenfeld, 2000, 2008). Particular attention was given to moments when participants utilized content knowledge to either make sense of a problem or to interpret student work. Participants' statements about their own understanding or about students' understanding were compared and constrated *within* cases and *across* the cases to generate and explore hypotheses about roles that content knowledge played in developing new knowledge.

The Ramp Problem

Here we focus on just one of the kinematics tasks used in the interviews. It was adopted from Trowbridge and McDermott's (1981) investigation of student difficulties with acceleration. In it, two balls roll down ramps starting from rest. Students are asked to compare the accelerations. Ball A gains 27 cm/s in 3 s (9 cm/s^2), while Ball B gains 30 cm/s in 4 s (7.5 cm/s^2):



The problem is particularly challenging because it invites two kinds of mistakes. One is to only consider the velocities. This can result from failing to distinguish between the concepts of acceleration and velocity. A second mistake is to calculate speed using v = x/t, and then acceleration using a = v/t. While this results in a correct comparison, the thinking behind it fails to distinguish between change in velocity and average velocity.

Although this exact kind of task is not as common in mathematics courses as it is in physics, students encounter ideas connected to velocity, acceleration, and various sorts of averages in differential calculus. In particular, with tasks such as this, many students do not clearly distinguish between the concepts of velocity and acceleration. Some will say that things that are moving fast are accelerating fast (even if they are moving fast with constant velocity). Confusing quantities and their rates of change is common across concepts and representations and is especially common in cases where the quantity itself is a rate of change, and in this case when they are tied to informal everyday language. Considerable research on student thinking about these concepts is found in the physics education research literature base but these and other derivative-related ideas are also known to be challenging for mathematics students (see, e.g., Zandieh, 2000).

Three Cases

Here we present data and analysis to illustrate that Jamie emerged from the activity of examining student work with richer knowledge of student thinking than was apparent at the start of the interview. We provide brief sketches of the other two cases where neither instructor appeared to develop PCK during the interview but one does appear to develop content knowledge.

Jamie Develops PCK

Jamie was the only participant who had little difficulty solving the ramp problem. After solving it, he was prompted to generate examples of student work. Jamie offered the following:

> So the first thing they might do is they might say that, acceleration is meters per second squared. And they might just be…well it's only being accelerated on this side [along the ramp], so I'm going to ignore what's on this side [along the flat track]… And I'm gonna say, this one is 40.5 over 3 times 3. And this one is 60 over 4 times 4. So, then I'd say like, 'A' is greater acceleration.

This quote illustrates how Jamie generated an example student solution, in part, by considering the units of acceleration as a piece of content knowledge a student might know and apply to the problem. While the student approach seems plausible, it is not actually a common student approach. The solution is, however, numerically similar to an approach actually taken by students. Students commonly use the ramp numbers to construct the same ratio that Jamie does, but students do so by substituting into x =v/t and a=v/t, not by considering the units.

When asked about what the approach would mean in terms of understanding, Jamie adds,

> [It] would mean that the student understands the units of acceleration, but that would be about it. [The student] doesn't understand how acceleration, velocity, distance, and time are inter-related, and how to go from those pieces of information.

Here we see that Jamie makes a rather broad diagnosis of the student difficulty, naming each of the four major kinematical quantities as among the things the student is having trouble relating. Based on these and other statements made by Jamie early in the interview, we describe Jamie's ideas about how students might solve the problem as neither accurate in terms of anticipating student approaches nor very specific in terms of diagnosis student difficulties.

Later in the interview, however, Jamie offers a very different decsription and diagnosis of the same student work:

> They are doing something different, because they are not finding a final velocity, which you have to use. They are finding an average velocity… So this is what this students' approach doesn't understand: the difference between change in velocity and the average velocity. What's missing from this student's understanding is that to find our acceleration, what we need is the change in velocity not the average velocity.

Without going into too much detail of the process by which Jamie changed his thinking about the student work, we highlight here several key differences between his earlier and later thinking. Earlier, Jamie characterized the student approach as taking the units, but he later characterizes this same student work in terms of finding an average velocity. This re-description is important in two ways. First, it brings Jamie's account in closer alignment with what students actually do when solving the problem. Second, Jamie's re-description of the student resulted from him bringing additional content knowledge to bear on the student work. Specifically, Jamie worked to re-express the student's calculation from units composed as "m/s^2" to units composed "(m/s)/s". He then recognized this second ratio as an expression containing an average velocity. In re-describing the student work, we see Jamie drawing on content knowledge about ratios and average velocity. Importantly, the object toward which Jamie applies this content knowledge became the student work itself, and in this phase of his work new content knowledge appeared that Jamie did not use while he solved the problem for himself.

In addition to changing his description of the student work, Jamie changes his diagnosis of the student difficulty. He originally located the difficulty quite broadly among many kinematics variables, but Jamie later locates the difficulty more specifically as being between change in velocity and average velocity. This is more specific in two senses. First, Jamie narrows the space of possible quantities the student is having trouble relating. Second, it is more specific with respect to physics content, with Jamie now specifying three different velocity-related quantities in his diagnosis. Finally, Jamie's diagnosis is also more *accurate* because it is more aligned with findings from the original research using the problem (Trowbridge and McDermott, 1981).

We claim that Jamie's statements made near end of the interview convey insights into student thinking that Jamie did not appear to know earlier in the interview. These insights emerged, in part, as Jamie sought to understand the nature of the student work and its validity. This process involved substantial considerations of the student work in terms of content knowledge.

Sam Develops Content Knowledge and Alex Struggles with the Problem

Sam's initial attempts to solve the problem included common student difficulties documented in research using this task. Sam did not explicitly consider the time it takes the ball to speed up. In addition, Sam states that the ball "reaches" an average velocity, when it is (arguably) more appropriate to speak of the ball reaching a final (instantaneous) velocity. In this sense, neither Sam's calculations nor ways of talking about the problem clearly distinguish between the concepts of average velocity and instantaneous velocity.

Later, after examining student solutions, Sam's thinking about the content is quite different and takes into account the amount of time the ball accelerates. More importantly, Sam's discussion of the problem becomes increasingly differentiated with respect to velocity concepts, distinguishing and relating instantaneous and change in velocity.

Like Sam, Alex struggled with the ramp problem. Alex did not to distinguish among different velocity concepts and, in essence, found an arithmetic difference between final velocity and average velocity, rather than a change in instantaneous velocity. Alex also viewed the sample student work through the framework that acceleration is "velocity" over "time" with it being possible to calculate different accelerations.

Discussion of Cases

The three cases illustrate the range of outcomes we observed during our investigation. While Jamie and Sam did articulate new insights during the interview, we cannot claim that these insights necessarily represent new stable forms of knowledge that would be brought to bear in other contexts. We are, however, still concerned with understanding *how* Jamie and Sam were able to learn something new from the activity of attending to student thinking We see in these cases the beginnings of processes that support stable knowledge development, and thus hope to better understand the conditions, contexts, and practices that enable it.

Based on our more extensive analyses, we hypothesize two specific processes that supported the beginning of such new knowledge development:

- First, for both Jamie and Sam, the activity of interpreting student work elicited content knowledge they had not used while solving the problem or generating student work.
- Second, Jamie and Sam developed specific and accurate interpretations of student thinking, which were then used as the basis for making more general claims.

In Jamie's case, we see a shift from describing the student work as "using units" to "taking an average velocity." This re-description required the application of knowledge about average velocity–knowledge that Jamie did not appear to use while solving the problem. Jamie was then able to leverage this interpretation to make a more general claim that the student work represents difficulty understanding the difference between average velocity and change in velocity. Jamie's case, thus, is an existence proof that even novice instructors (Jamie was a first time instructor) may be able to develop teaching-related knowledge via activities of attending to student work.

In Sam's case, we also see content knowledge used to interpret student work, Sam eventually made key content connections while examining an incorrect piece of student work. Based on this insight, Sam was able to generate a more general statement about what the error in his own understanding had been. Alex, on the other hand, utilized the same content knowledge when solving the problem and interpreting student work. Ultimately, the nature of this knowledge was not only insufficient for understanding the problem but also insuffficent for making progress in understanding the nature of the student difficulty.

Conclusions and Implications

These cases provide some insights into interactions that can occur between content knowledge and knowledge of student thinking as instructors engage in teaching-related tasks. Participants not only interpreted student work through lenses of their content knowledge, but those interpretations were (for some) generative of new knowledge. For Jamie, strong content knowledge enabled him to emerge with new knowledge of student thinking. Sam was able to develop new content knowledge but Alex's content knowledge did not seem to support new knowledge development. These kinds of fine-grained examinations of knowledge contribute to the research community's understanding of interactions and inter-dependencies of different types of knowledge (e.g., content and PCK). This builds on work such as that of Sherin (2002) where influences of types of knowledge on one another were found to shape how teachers implemented novel curricular materials. Findings shed light on how teachers learn while engaged in the work of teaching and could be used to inform the design of professional development activities to support such learning for novice teachers. In particular, findings suggest that it may be productive (and necessary) to provide teachers with opportunities to strengthen their knowledge of the content as part of professional development. It appears that without strong content knowleldge there may be some apects of knowledge of student thinking that teachers are unable to develop from their "on-the-job" teaching experiences.

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