EFFECTS OF COLLABORATIVE REVISION ON BELIEFS ABOUT PROOF FUNCTION AND VALIDATION SKILLS

Emily Cilli-Turner
University of Illinois at Chicago

Although there is much research showing that proof serves more than just a verification function in mathematics, there is little research documenting which functions of proof undergraduate students understand. Additionally, research suggests that students have difficulty in determining the validity of a given proof. This study examines the effects of a teaching intervention called collaborative revision on student beliefs regarding proof and on student proof validation skills. Student assessment data was collected and interviews were conducted with students in the treatment course and in a comparison course. At the end of this study, we will produce a categorization of the proof functions that students appreciate, as well as a determination of the value of the teaching intervention on students' abilities to correctly classify proofs as valid or invalid.

Key words: proof validation, transition to proof, collaborative learning, function of proof

Introduction

This study aims to investigate the effects on beliefs about proof and argument validation skills of students in an introduction to proof course by employing a process called collaborative revision in the classroom. *Collaborative revision* refers to the process in which students present a proof they have written to their classmates and the other students are encouraged to make comments and point out inconsistencies in order to ensure that the proof is valid. Based on feedback from classmates, the student then revises the proof and presents it again, repeating the process until the proof is valid and includes all the relevant details. In this context, this project aims to answer the following research questions:

- What are students' beliefs about the function of proof in mathematics and how do these beliefs change during a course using a collaborative revision teaching intervention?
- How does collaborative revision affect students' proof validation skills?

Background & Related Literature

The role of proof as useful for verification of a statement has been well documented (e.g. Harel, 2007; Mason et al., 1982). However, researchers have categorized many other functions of proof in mathematical practice. Hanna (1990) and deVilliers (1990) suggest that in some cases proofs can be *explanatory* and provide insight as to why a certain statement is true, especially when a conjecture was arrived at by empirical results. DeVilliers (1990, 2002) also argues for four more functions of proof in mathematics. The first is that of the use of proof as a *means of discovery* in that proof can sometimes lead to new results in a field. The second is that proofs serve an important *communicative* role, since they are the main way mathematical knowledge is transmitted. Third, proof has the function of being an *intellectual challenge* in that the completion of a proof can be very satisfactory for many mathematicians. Finally, proof can expose logical relationships between statements and can serve as a tool for *axiomatizing* results in a mathematical system.

Furthermore, Weber (2002) claims that proof can serve as the *justification for a definition* in mathematics or, in the context of teaching and learning, proof can *illustrate techniques* in advanced mathematics. Yackel and Cobb (1996) note that another role of proof can be to *provide autonomy* to students by allowing them to create their own mathematical knowledge.

Indeed, Rav (1999) proposes (and Hanna & Barbeau (2009) agree) that proofs are of the utmost importance in mathematics, since they, instead of theorems, are the main vehicles in which mathematical knowledge is contained and transferred.

If proofs are the bearers of mathematical knowledge, the ability to determine if a given proof is valid is an important skill. Selden & Selden (2003) highlight that this skill is invaluable for not only future mathematics educators because they will someday have to evaluate student proofs for assessment purposes, but also for future mathematicians because they will have to examine others' proofs to learn about new mathematics that is being produced. Additionally, proof validation is intricately linked to proof construction so students in transition to proof courses need both skills (Selden & Selden, 2003).

How then should these important concepts be taught to undergraduate students? There is much research to support the hypothesis that collaborative learning can greatly enhance student learning of appropriate mathematical proofs. Yackel & Cobb (1996) note that participation in a community of learners can be a vital part of students' success in mathematics and learning can be regarded as a relationship between communal classroom processes and individual activity. Additionally, in a study, Strickland & Rand (2012) allowed students to submit multiple revisions of proofs in response to teacher feedback and measured the effects on student learning. The teacher comments given were minimal, often just circling a confusing or incorrect passage of the proof, and students were allowed as many revisions as needed. Although the data set was small, on average, students in the revision group did better on the final exam. Thus, collaborative revision is a way to explore the benefits of combining these proven techniques and this study examines the effects on students in an introduction to proof course using a collaborative revision teaching intervention in the classroom.

Methodology

Participants were drawn from two lecture-based transition to proof courses at a large Midwestern university and from a course designed to supplement these courses, where the teaching intervention was enacted. A comparison group was desired to determine the effects of the teaching experiment when compared to a lecture-based course, but the researcher could only get access to the supplementary course, thus we only have a comparison course, not a proper control.

The 43 participants were given a pre and post assessment designed to evaluate how students think about the function of proof and if students can identify valid proofs. The first question on the assessment, modeled on a study by Healy & Hoyles (2000), is an open response question asking for general thoughts on the purpose of proof in mathematics, phrased as "What do you believe is the purpose of proof in mathematics?" The purpose of this question is to determine which functions of proof students recognize and how important they believe proof to be in mathematics. The same question was asked on both the pre and post assessment to gauge how students' beliefs about proofs change during the course. Additional questions on the assessment concerned the role of proof and statements were given about the myriad functions of proof from the literature, as discussed above. Students were asked to rate their agreement with each statement on a five point Likert scale, from strongly agree to strongly disagree. The purpose of these questions was determine if students consider other functions of proof besides verification and if students believe that some functions of proof are more important than others.

Another part of the assessment required the students to examine four correct and incorrect 'proofs' of a given statement and determine whether each was a valid or invalid proof. The proofs presented to the students are adapted for this study from the proofs given to high school students in Healy and Hoyles (2000) to be appropriate for undergraduates and did not require outside concepts or theorems that students may not remember from a previous course.

Students were first asked to determine if each proof is valid or invalid. The terms valid and invalid are intentionally not defined in order to see if students gain more of an understanding of what a valid proof entails throughout the course of the semester. There were also two other questions asking students, on a three-point Likert scale, how well they feel they understand the proof and how certain they are about their classification.

Student interviews were also conducted with six students (three in the treatment course and three in the comparison course) to allow students to elaborate on their classifications of proofs as valid or invalid and assess if these skills improved in students in the treatment course as compared with students in the other course. Students from the treatment and comparison courses were interviewed individually twice during the course of the semester, once shortly after the pre-assessment and again shortly after the post assessment. These were semi-structured interviews and questions asked to students, according to Zazkis and Hazzan (1998), are performance questions, unexpected "why" questions and reflection questions, where students were to explain their thought processes during the proof validation task on the assessment.

Data Analysis Plan

A quantitative analysis of the questions about proof function evaluated on a Likert scale will be performed to determine which functions students consider most important. Statistical analysis will be completed to determine any significant difference between students pre and post assessment scores, as well as between the treatment and comparison courses. A qualitative analysis of students' responses to the question "What do you believe is the purpose of proof in mathematics?" will be performed using the functions of proof identified in mathematics education literature outlined above as starting categories and adding other categories as needed.

The framework for data analysis of the interviews will be developed iteratively. I will begin with the framework of Harel & Sowder's (1998) proof schemes categorization and data from the interviews will be used to inform the framework and in turn the framework will help classify students into a proof scheme category. Preliminary findings support using this framework as the students seemed to clearly fall into one of the proof scheme categories.

Questions for the Audience

- How can collaborative revision be implemented so that it is balanced with lecture in a transition to proof course?
- What are other implications that you see from this work?
- This work is part of a larger study. What other data do you think would be helpful to collect to fully analyze the collaborative revision process?

References

deVilliers, M. (1990). The role and function of proof in mathematics. *Pythagoras, 24*, 17–24.

deVilliers, M. (2002, October). *Developing understanding for different roles of proof in dynamic geometry*, Paper presented at ProfMat 2002, Visue, Portugal.

Hanna, G. (1990). Some pedagogical aspects of proof. *Interchange, 21*(1), 6–13.

Hanna, G., & Barbeau, E. (2009). Proofs as bearers of mathematical knowledge. In G. Hanna, H. Jahnke, & H. Pulte (Eds.), *Explanation and proof in mathematics: Philosophical and educational perspectives* (pp. 85–100). New York, NY: Springer.

Harel, G. (2007). Students' proof schemes revisited. In P. Boero (Ed.), *Theorems in school*, pp. 65–78. Rotterdam, The Netherlands: Sense Publishers.

Harel, G. & Sowder, L. (1998). Students' proof schemes: Results from exploratory studies. *CBMS issues in mathematics education, 7*, 234–283.

Healy, L. & Hoyles, C. (2000). A study of proof conceptions in algebra. *Journal for research in mathematics education, 31*(4), 396–428.

Mason, J., L. Burton, and K. Stacey (1982). *Thinking mathematically*. London, UK: Addison-Wesley.

Rav, Y. (1999). Why do we prove theorems? *Philosophia mathematica*, *7*(1), 5–41.

Selden, A. & Selden, J. (2003). Validations of proofs considered as texts: Can undergraduates tell whether an argument proves a theorem? *Journal for research in mathematics education, 34*(1), 4–36.

Strickland, S. & Rand, B. (2012). Improving undergraduates' novice proof-writing: Investigating the use of multiple drafts. In S. Brown, S. Larsen, K. Marrongelle, and M. Oehrtman (Eds.), *Proceedings of the 15th Annual Conference on Research in Undergraduate Mathematics Education*, Portland, Oregon, pp. 555–559.

Weber, K. (2002). Beyond proving and explaining: Proofs that justify the use of definitions and axiomatic structures and proofs that illustrate technique. *For the learning of mathematics, 22*(3), 14–17.

Yackel, E. & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for research in mathematics education, 27*(4), 458–477.

Zazkis, R. & Hazzan, O. (1998). Interviewing in mathematics education research: Choosing the questions. *Journal of mathematical behavior, 17*(4), 429–439.