PERFORMANCE AND PERSISTENCE AMONG UNDERGRADUATE MATHEMATICS MAJORS

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There is little mixed methods research into the patterns of course taking, performance, and persistence among mathematics majors, in general, and among secondary mathematics majors, in particular. Drawing from a sample of 42,825 mathematics enrollment records at two universities over a six-year period, this study presents quantitative summaries of mathematics majors' performance and persistence in undergraduate mathematics courses alongside qualitative themes from interviews of nine secondary mathematics majors at one of the universities. Implications include potential strategies for mathematics programs and faculty to support the success of mathematics majors in undergraduate mathematics coursework, with special emphasis on prospective secondary mathematics teachers.

Keywords*:* mathematics performance, persistence, mathematics majors, preservice teachers

A recent U.S. federal advisory panel report recommended expanding mathematics instruction by faculty from other disciplines (e.g., physics or engineering) and building "a new pipeline for producing K-12 mathematics teachers from undergraduate and graduate programs in mathematics intensive fields other than mathematics." (PCAST, 2012, p. vii). Citing mathematics as a bottleneck in STEM education, the report suggests students in a range of STEM programs may be better served by learning the specialized mathematics needed for their fields from faculty in those fields, views held in direct contrast of many mathematicians (AMS, 2012). In the context of the mathematics preparation of teachers, the PCAST recommendations pose a serious challenge to the primary role of university mathematics faculty in the preparation of secondary mathematics teachers.

Nearly all undergraduate teacher education programs build prospective secondary mathematics teachers' content knowledge through courses in an undergraduate mathematics major, including courses such as single- and multi-variate calculus, differential equations, linear algebra, introduction to proof, statistics, and abstract algebra (Conference Board of Mathematical Sciences, 2012) . Though research is unclear on whether advanced undergraduate mathematics courses necessarily improve the specialized knowledge of mathematics needed in middle and high school classrooms (Hill et al., 2005; Kahan et al., 2003), U.S. secondary mathematics teachers typically complete at least four times as many courses in mathematics as they do in education (Monk, 1994).

Mathematics majors collectively consist of just 1% (12,363 of 1,119,579) of bachelor's degree earners in the U.S. (Lutzer, 2002), and there has been limited research on the patterns of academic outcomes among the even smaller number of students concentrating in secondary mathematics teaching. Existing research does, however, suggest earning a bachelor's degree in mathematics can be a difficult experience. Mathematics courses have the second lowest grade distribution among all majors (Rask, 2010), and more than 3 of 5 students who declare a mathematics major switch out of the major (Seymour & Hewitt, 1997). Moreover, students' early work toward earning mathematics degrees may be complicated by the role of mathematics courses as a filter for business and STEM programs (Moore & Shulock, 2009) and a variety of social factors, such as a lack of family support and the commonly held societal view of mathematics as a male domain (Ma & Kishor, 1997). Secondary mathematics majors may also face the challenge of navigating between grading

norms in mathematics courses and those established in education courses, which have the highest grade distribution among all college majors (Rask, 2010).

Though learning undergraduate mathematics is central to learning to teach mathematics (Cooney & Wiegel, 2003), there has been limited research specifically addressing the performance of prospective secondary mathematics teachers in undergraduate mathematics courses. This may be due in part to small enrollment numbers. Students majoring in secondary mathematics are typically outnumbered in nearly all required courses by STEM majors outside of mathematics (Lutzer, Rodi, Kirkman, & Maxwell, 2007). In 2005, just 3,400 secondary mathematics majors joined about 15,000 applied or liberal arts mathematics majors among 699,000 students enrolled in undergraduate mathematics courses at the calculus level or beyond (Lutzer et al.). There have been investigations into the subject matter knowledge gained by secondary mathematics majors through undergraduate mathematics programs (e.g., Bryan, 1999; Even, 1993), there is need for more research on mathematics performance and persistence.

The research questions address (a) the patterns of course enrollment and letter grades earned by mathematics majors in mathematics courses, and (b) how secondary mathematics majors experience and respond to difficulties in undergraduate mathematics courses:

1. (Quantitative) What characterizes mathematics majors' academic performance and persistence in undergraduate mathematics courses at two universities?
2. (Qualitative) What describes prospective secondary mathematics teachers' perceived performance in undergraduate mathematics courses at one university?

Methods

The two universities included in the quantitative strand are similar in many ways – each is a minority serving public institution located in the same central U.S. state, enrolls about 7,500 full-time undergraduate students, admits about 90% of applicants (many of which are community-college transfers), and offers undergraduate and master's programs in mathematics. The main contrasts come from the facts that University A is a regional campus in a mid-sized city, while University B is in a large metropolitan area and enrolls mostly female students. Data collection consisted of institutional academic records from both universities and face-to-face interview and survey data from University B. Quantitative data included student, course, and performance variables for all students enrolled in one or more mathematics courses during the study period of six academic years (fall 2005 to spring 2011). For each enrolled mathematics student, the institutional records included unique student identifier, university, ethnicity, sex, ACT/SAT mathematics score, high school grade point average, age, undergraduate major, academic level, course name, course section, instructor, term, and final letter grade.

The qualitative strand of the study employed semi-structured interviews crafted after the protocol of Seymour and Hewitt's (1997, p. 401) large-scale study of switching- and non-switching undergraduate science, mathematics, and engineering majors. Interview questions invited study participants to discuss experiences surrounding choosing a college major, performance in high school and college mathematics, exam and course grades in mathematics, quality of mathematics instruction, sources of academic support, (self- and peer) experiences of major switching, and career plans. As part of the interview protocol, participants completed a modified version of the 30-item Mathematics Self-Efficacy and Anxiety Questionnaire (May, 2009) and self-reported their mathematics self-efficacy (Bandura, 1997) in the content of each of their undergraduate mathematics courses on a scale of 0 (not confident) to 10 (very confident).

The quantitative data sample initially included all $N = 42,825$ enrollments in mathematics courses by undergraduate students at the two universities during the six-year

study period. While this larger sample allowed for analysis of course enrollment patterns, measures of persistence in mathematics were focused on a subsample of n = 12,522 mathematics course enrollments by students who completed all their mathematics classes during the six-year study period. Demographics in the overall sample suggests the data set included more female (72%) than male (28%) students, and no majority student ethnicity (48% White, 31% Hispanic, 14% Black, 4% Asian, 2% Other). In addition, the sample included comparable enrollments by Freshmen (27%), Sophomores (17%), Juniors (20%), and Seniors (26%). Most students were either between the ages of 17 to 22 (48%) or 23 to 27 (35%), with just 16% older than 27 years old.

The nine qualitative interview participants were purposefully sampled at University B. All undergraduates from University B in the persistence subsample who had previously declared a secondary mathematics major (approximately 40 students) were invited to participate in the study, so the interview participants represent a self-selected group of secondary mathematics teachers interested in sharing their experiences in mathematics courses. Eight of the participants were still mathematics majors at the time of the interview, and six participants planned on working as a teacher after completing their degree. The numbers of mathematics enrollments were 4, 10, 10, 12, 12, 13, 18, 19, and 27, respectively.

Results

Performance of Mathematics Majors in Mathematics Courses

Mathematics majors represented 1.9% of the unique students who took one or more mathematics courses during the study period. However, enrollments by mathematics majors accounted for 5.8% of all the mathematics enrollments, and at least one mathematics major completed every one of the 28 mathematics course titles offered during the study period. Mathematics majors were found in the highest percentages in classes with the lowest enrollments. For example, mathematics majors formed just 0.8% of the enrollments in the 8 introductory courses (e.g., College Algebra) which account for 77% (33,177 of 42,825) of the combined mathematics enrollments. Mathematics majors formed less than 10% of enrollments in Trigonometry, Precalculus, and Calculus I, and less than a third of enrollments in Probability & Statistics, Calculus II, and Discrete Math I & II. Mathematics majors were the minority in 6 of the 10 most taken courses by mathematics majors, including Probability & Statistics, Calculus I & II, Discrete Math I, Linear Algebra, and Introductory Statistics I.

Though mathematics majors were among the least common major types enrolled in mathematics courses (only "Undeclared" had lower absolute enrollment numbers), mathematics majors earned the highest overall distribution of letter grades. Forty-eight percent of mathematics majors maintained an average letter grade of B or better in their mathematics courses; the combined mean mathematics GPA of mathematics majors was 2.5 (*Mdn* = 2.9, *SD* = 1.3, Range = 0 to 4). As indicated in Table 1, more than one-in-three enrolled mathematics majors earned a letter grade of A (highest among all major types), and the overall DFWI rate for mathematics majors was 20.8% (lowest among all major types). However, six course titles showed DFWI rates among mathematics majors of the courses of more than 25%, including Abstract Algebra (32%), Introduction to Proof (32%), Linear Algebra (30%), Probability & Statistics (29%), Real Analysis (28%), and Calculus I (26%).

Table 1. Major types among undergraduate mathematics courses at two universities.

Major Type	%A	%B	%C	%D	%F	%W	%I	Total
Health Sciences	32.6	24.3	18	6.6	11.5	5	2	*11,780*
Liberal Arts	20	23.8	21.4	7.5	14.5	8.7	4.1	*8,843*
Sci/Engineering	23	23	22.1	9.9	12.6	8.1	1.3	*7,248*

Business	25.6	24.9	21.5	7.4	10.4	8	2.2	*6,531*
Education	22.7	24.1	21.6	7.4	12.6	6.9	4.6	*5,102*
Mathematics	34.9	26.4	17.9	6.9	8.5	4.7	0.7	*2,249*
Undecided	22.8	29.3	22.8	10.1	11.7	3.4	0	*386*
Combined	26.1	24.2	20.4	7.6	12.1	7	2.6	*42,139*

Persistence among Mathematics Majors in Mathematics Courses

The subsample of mathematics course enrollments by students who completed all their mathematics courses during the study period (n = 12,522) included just 98 unique students (0.8%) who declared a mathematics major during the study period. The mean number of mathematics courses attempted by mathematics majors was 5.0 (*Mdn* = 3, SD = 4.5, Range = 1 to 19), and 69.4% of mathematics majors took 5 or fewer mathematics courses. However, just 29.4% of mathematics majors passed 6 or more mathematics classes with a letter grade of C or better (*M* = 4.3, *Mdn* = 3, SD = 4.4, Range = 1 to 19). More than half (54.1%) of mathematics majors attempted 3 or fewer mathematics classes, 15.3% attempted 4 or 5 classes, 12.3% attempted 6 to 9 classes, and 18.4% attempted 10 or more classes. Based on the enrollment numbers, we estimate only about one in four mathematics majors completed enough mathematics courses to fulfill undergraduate degree requirements.

Overall, 35.7% of mathematics majors failed at least 1 mathematics class, including 39.3% of mathematics majors who enrolled in 7 or more mathematics classes. There were 8 courses for which 10% or more of the enrolled mathematics majors had previously attempted the course, including Introduction to Proof (19.3%), Abstract Algebra (16.0%), Real Analysis (15.2%), Probability & Statistics (14.7%), Calculus I (12.2%), and Linear Algebra (10.1%). Collectively, 9.5% of mathematics majors enrolled in mathematics courses were retaking the course, with 1.3% attempting the course for a third, fourth, or fifth time.

Perceived Performance in Mathematics Courses among Secondary Mathematics Majors

The major themes in our qualitative interviews included struggles described by mathematics majors in mathematics courses and the diverse ways in which some of the students worked to overcome those obstacles. In all, we found eight areas identified by multiple students as sources of personal frustration in mathematics courses, including the course instructor (5 participants), family/parenting (4 participants), difficult content (4 participants), inadequate understanding of prerequisite content (3 participants), online course format (3 participants), difficulty in understanding the textbook (2 participants), and a perceived lack of real-world applications (2 participants). All of the study participants described substantial struggles in at least one of their undergraduate mathematics courses. Five participants failed to pass (with a C or better) at least one required mathematics course on their first attempt, and three of the participants took a specific mathematics course three or more times. Consistent with Champion (2010), these participants described performing well as elementary and secondary school mathematics students and recalled rarely struggling to learn new mathematical ideas prior to college. In all cases except Goldie, participants described first encountering substantial difficulties in a mathematics courses while in college.

Though each student's stories of struggles were unique, Kirsten's description of her undergraduate mathematics coursework exemplifies the extent to which self-perceived performance had deep impacts on mathematics majors' lives. Kirsten excelled in high school mathematics courses and entered college planning to become a high school mathematics teacher. After earning high grades in the required lower-division mathematics classes, Kirsten described first encountering major struggles when she earned a D in Real Analysis:

> I had a complete lack of understanding [of Real Analysis]…. I had to retake it because it was a major class, but I just, I couldn't go back to school. I didn't know what I wanted to do with my life. That D really stung me a lot. So…. for the fall, I substitute

> taught, and that was important in getting me back to school, because I did a lot of substitute teaching and I liked the school.

Kirsten's poor performance in Real Analysis led her to temporarily withdraw from the university and return home. She reenrolled a semester later and devoted a significant amount of time and effort to Abstract Algebra:

> I worked myself to death …. This is what I call my crazy time. I got so depressed after that class that I actually started seeing a psychologist…. I took Analysis again. I took Abstract Algebra again. I took a Geometry class…. I kept on trying, thinking I would maybe just be done, but I was emotionally shot at this point.

After failing Abstract Algebra, Real Analysis, and Geometry, Kirsten formally withdrew from the university and took two years to work and seek treatment for clinical depression. She attributed her mathematics hardships as major sources of her depression, recalling, "I'm struggling with math classes. How can I even graduate? How can I become a math teacher? Is this the right path? What am I supposed to do with my life? What do I do now?" Before returning to school, Kirsten described having to work through a fear of failure in mathematics with a therapist. She eventually came to believe, "I probably am capable of graduating college. I am capable of this… Just because I failed once or twice doesn't mean that I will continue to fail in every other thing." So, three years after leaving the university, Kristen moved back to her home state and enrolled part-time in one mathematics course at University B. The next semester she took two courses (including Abstract Algebra), and after earning an A in all her classes, described plans to return as a full-time student in the fall. She was unsure of whether or not she still wanted to teach high school mathematics, but confidently expected to complete her undergraduate mathematics degree in the near future.

Though seemingly extreme, it was unclear whether Kirsten's reports of intense and interconnected personal and academic struggles in mathematics courses were atypical. Three participants described intense feelings of hopelessness in calculus courses as they worked to overcome perceived deficiencies in understanding of pre-requisite content. Four participants recalled struggling to catch-up in classes following personal crises, including caring for children, grieving the death of a close relative, and even having a child mid-way through an Abstract Algebra course.

Discussion

Several aspects of the study findings (e.g., interview themes, grade distributions of mathematics majors) should be considered exploratory and tied to the context at the two research sites. Nonetheless, the findings suggest several insights into how mathematics programs can help secondary mathematics majors succeed in an undergraduate mathematics programs. The statistical results suggest (1) mathematics majors are among the best performing students in mathematics courses and (2) many mathematics majors choose not to persist in undergraduate mathematics. Fewer than half of mathematics majors took more than three mathematics courses, and nearly two in five mathematics majors who completed seven or more mathematics classes failed at least one course. The results suggest that mathematics majors, though typically successful in mathematics courses, often experience significant difficulties with at least one mathematics course which may lead them to reconsider their choice of the major.

The quantitative and qualitative data also support the common sense claim that students' struggle most in mathematics classes with unfamiliar content, especially abstract and proof-based courses. Our interviews suggested these struggles may be amplified when students struggled early in course, perceived instructors to have strict expectations, or faced assignments that deemphasized computational procedures. Some of these struggles may be unavoidable, and with proper support, challenging courses can provide excellent

opportunities for learning. Well-designed courses like Introduction to Proof, Abstract Algebra, and Real Analysis can deeply extend mathematics majors' understanding, and can serve the especially important function of recalibrating the beliefs of secondary teaching majors who, prior to such courses, may think of mathematics in terms of memorization and assume the role of mathematics teachers is to lecture (Cooney, Shealy, & Arvold, 1998). Much of the gap between mathematics majors' history of success in mathematics courses and their struggles in challenging courses may well be bridged by clear communication of expectations along with formal and informal support structures.

The interview data also suggests approaches faculty might consider in order to help mathematics majors be successful in mathematics classes. For example, faculty might consider explicitly describing the qualitative meaning students can ascribe to exam scores (e.g., "a score above 70% on this test means you're on track"). Instructors might also use group quizzes or in-class activities to help mathematics majors make normative judgments of their understanding in relation to that of their peers. Fukawa-Connelly (2012) has recently outlined a way in which abstract algebra students can reach deep understanding of proofs by engaging in a student-centered classroom atmosphere in which students present and defend their proofs, and "high discourse communities" (Imm & Stylianou, 2012) can help students' understandings become the focus of class discussions and could help break the cycle of procedural-based instruction that makes it difficult for some secondary mathematics teachers to shift to conceptual-based teaching approaches (Cooney, Shealy, & Arvold, 1998).

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