OPPORTUNITY TO LEARN FROM MATHEMATICS LECTURES

Emilie Wiesner, Ithaca College
Timothy Fukawa-Connelly, University of New Hampshire
Aaron Weinberg, Ithaca College

Many mathematics students experience proof-based classes primarily through lectures, although there is little research describing what students actually learn from such classroom experiences. Here we outline a framework, drawing on the idea of the implied observer, to describe lecture content; and apply the framework to a portion of a lecture in an abstract algebra class. Student notes and interviews are used to investigate the implications of this description on students' opportunities to learn from proof-based lectures. Our preliminary findings detail the behaviors, codes, and competencies that an algebra lecture requires. We then compare those with how students behave in response to the same lecture with respect to sense-making and note-taking, and thereby how they approach opportunities to learn.

Keywords: opportunity to learn, implied observer, note-taking, abstract algebra

Many mathematics students experience proof-based classes primarily through lectures (Mills, 2012). Preliminary research indicates that the demands of learning from a live presentation of material, through the instructor's writing, speech, and gestures, are significantly more taxing than learning from a textbook (Fukawa-Connelly, Weinberg, Wiesner, Berube, & Gray, 2012). The goal of our research is to develop and use a framework to describe the opportunities to learn that are available to students in a proof-based mathematics lecture. Focusing on note-taking as one of the primary activities that students engage in during a lecture, our research questions are to identify:

1. What discrepancies between student notes and lecture content are common and under what circumstances do they occur?
2. How and to what extent do such discrepancies represent a missed opportunity to learn?

Lecture Content

Understanding what a student might learn from a lecture requires a description of the lecture itself. Here we briefly present a framework for describing the lecture in terms of its mathematical content and the way it is presented. We also use the idea of the "implied observer" to analyze the demands that the lecture places on the student.

Lectures contain numerous *mathematical components* that students must attend to and interpret including proofs, definitions, statements of theorems and algorithms, examples, and exposition. In addition to their mathematical aspect, these components can also be distinguished by their mode of presentation: written, spoken, and gestural. Lectures also have a temporal-spatial aspect: for example, a lecture is given only once, with limited time and space for writing; instructors may write notes in a non-linear fashion; or they may structure their board-work to make distinctions or connections between ideas.

The *implied observer* provides a description of what is required of an observer of a lecture in order to respond to the lecture in a way that is both meaningful and accurate (Fukawa-Connelly, Weinberg, Wiesner, Berube, & Gray, 2012; Weinberg & Wiesner, 2012); it is created by the lecture itself, as opposed to the intentions of the instructor or the actual observer (i.e. the student's own behaviors, codes, and competencies). The implied observer of a lecture can be characterized by a set of codes, behaviors, and competencies. A *code* is the implied observer's method of ascribing meaning to particular lecture content. The *competencies* of the implied observer are the knowledge, skills, and understandings that are required to understand the lecture. Finally, *behaviors* are actions—often mental actions—that the implied observer takes.

Opportunities for Learning

The National Research Council has defined the opportunity to learn as "circumstances that allow students to engage in and spend time on academic tasks…" (p. 333). Describing the lecture content and the associated implied observer is a necessary precondition to understanding the opportunities a student may have for learning from a lecture. In particular, we view students' opportunity to learn mathematics from a lecture as the interface between the implied observer and the actual observer. In this study, we explore how students react to different points in the lecture and analyze their reaction through the lens of the implied observer.

Data Collection and Analytical Methods

We have collected and are in the process of analyzing data in a pilot study. The participants are six mathematics majors who were enrolled in a standard abstract algebra class. The data corpus consists of videotaped classroom observations, videotaped interviews with students, and the students' notes. The class was videotaped 6 times over the course of the semester. The video was transcribed to include the written, spoken, and gestural components. In this way, the class observations were designed to capture as much of the "text" of the lecture as possible and were used to create a description of the implied observer.

After each observed class period, we collected the participants' notes and identified discrepancies between the students' notes and the "text" of the lecture. Interviews with students after each recorded lecture included showing video clips of the lectures and asking questions about their decision-making. We used the interviews, along with the notes, to help identify the behaviors, codes and competencies of the actual observers. To analyze the data we first read the interview transcripts, making comments indicating the behaviors, codes and competencies that students showed (Strauss & Corbin, 1994). In this proposal we focus on a "chunk" of one class meeting because of its role in motivating a subsequent proof and because multiple students omitted portions of the lecture content from their notes. We are currently attempting to describe to what extent these discrepancies represent a missed opportunity to learn and how they may be explained by differences between the implied and actual observers.

Results and Analysis

Summary of the class. For analysis, we consider a segment in class in which Dr. P asked the question, "What is Q?" meaning, the rational numbers. He noted how they had traditionally described the rational numbers as 'a over b' where a and b are integers and b is non-zero, but said that it was insufficient. Dr. P explained that because they had two elements, it made was reasonable to write the rational number as an ordered pair, or cartesian product. He then stated that a/b would be written as (a,b). Dr. P then noted that ¾ = 9/12 but (3,4) =/ (9,12) although they, meaning the class, would want it to be. Dr. P continued: "Some of these ordered pairs should be related to each other and others not. Now, on what basis do we say this is the same?" He claimed that they should have a way to test when ordered pairs are the same, and noted that cross-multiplication was perhaps the easiest way to test, although reducing to lowest common terms could also work, just that it would require "fiddling." He then defined the relationship a/b = c/d when ad=bc and proceeded to prove that it was an equivalence relation that gave the desired results [Dr. P's board work is included in Appendix 1].

This "chunk" is dense with codes, competencies, and behaviors. For example, there are a variety of symbolic codes embedded in the board work, including the set notation used to defined Q, the notation ZxZ-{0} to represent ordered pairs, and the double arrow indicating equivalent statements. The implied observer has competencies encompassing knowledge of equivalent fractions and an understanding of what makes a well-defined equivalence in mathematics. Behaviors include responding to Dr. P's rhetorical questions by thinking about what difficulties are posed by the familiar definition of the rationals and how the example 3*12=4*9 could be generalized.

We also note that much of Dr. P's presentation was spoken and not written. This potentially widens the gap between the implied and actual observers, as it may be more difficult for students to call up necessary codes and competencies or to enact required behaviors.

Students' note-taking decisions. Students' notes on this segment of class contain a variety of omissions of the written lecture content [See Appendix 1], generally focused on the specific example (3,4) ≠ (9,12). Only 1 student's notes, Jocelyn's, contained additional writing related to the spoken lecture content. These discrepancies between the lecture content and students' notes reflect a variety of student choices.

1) S*ome students omit while also making appropriate mathematical interpretations.* Ted's general note-taking strategy was to write only definitions. In keeping with this, he did not record any part of this segment in his notes. However, in an interview he was able to articulate the dilemma that Dr. P was indicating and how it would be resolved.

2) *Students omit when they think they understand but may not have a complete understanding.* During her interview Petra indicated that she had recorded that (3,4)=/(9,12), and claimed the goal of the lecture was to have them be equal. However, she did not write down the step, "3*12 = 4*9" because, "It was easy... I know, I know why those are equal." This appears to reflect a lack of understanding of the purpose of the example, as she states, "I don't know why [the ordered pairs] have to be related."

3) *Students omit when they don't understand and think they may be confused later.* Meredith's notes do not reference the example. In an interview, Meredith said, "I remember him writing that and saying like, yeah that should be true, and him saying it wasn't. And then I understood that the points weren't the same, but then I didn't understand, I guess the bigger

concept of why the whole thing didn't, so I didn't write it. Because I think it would just confuse me looking back at it."

Discussion

Students' stated reasons for omitting lecture content suggest a variety of gaps between the implied and actual observers. Moreover, these omissions reflect not only a lack of understanding during class but may also limit their opportunity to learn from their notes outside of class. While we believe that our method will produce interesting results, we are cautious in our assertions given that our analysis is ongoing. We believe that the major contribution will be to articulate a method to describe, on a minute-by-minute basis, what a particular student has the opportunity to learn from an undergraduate lecture in a proof-based mathematics class.

Questions we intend to discuss

1) How effective is the implied reader framework at capturing students' opportunity to learn? What aspects of opportunity to learn does this fail to address?
2) To what extent are students' notes an effective tool for investigating opportunity to learn?

References

Fukawa-Connelly, T. Weinberg , A., Wiesner, E., Berube, S., & Gray, K. (2012). *Student Note Taking Behavior in Proof-based Mathematics Classes.* Proceedings of the SIGMAA RUME*: Conference on Research in Undergraduate Mathematics Education*. Portland, OR. http://sigmaa.maa.org/rume/crume2012

Mills, M. (2012, February). Investigating the Teaching Practices of Professors When Presenting Proofs: The Use of Examples. Conference on Research in Undergraduate Mathematics Education, Portland, OR.

National Research Council (NRC). (1996). *From analysis to action: Undergraduate education in science, mathematics, engineering and technology*. Washington, DC: National Academies Press.

Strauss, A., & Corbin, J. (1994). Grounded theory methodology: An overview. In N. K. Denzin & Y. S. Lincoln (Eds.), *Handbook of qualitative research* (pp. 273 – 285). Thousand Oaks: Sage Publications.

Weinberg, A. & Wiesner, E. (2011) Understanding mathematics textbooks through reader-oriented theory. *Educational Studies in Mathematics 76* (1), 49-63.

Appendix 1

What Dr. P's board work and students' notes.

$\mathbb{Q}=\{\frac{a}{b} \mid a,b\in\mathbb{Z},\ b\neq 0\}$ $\frac{a}{b}\longleftrightarrow(a,b)\in\mathbb{Z}\times(\mathbb{Z}-\{0\})$ $\frac{3}{4}=\frac{9}{12}$ $(3,4)\neq(9,12)$ **Dr. P:** $3\cdot 12=4\cdot 9$	Jocelyn: $\mathbb{Q}=\{\frac{a}{b} \mid a,b\in\mathbb{Z},\ b\neq 0\}$ $\frac{a}{b}\longleftrightarrow(a,b)\in\mathbb{Z}\times(\mathbb{Z}-\{0\})$ $\frac{3}{4}=\frac{9}{12}$ $3\cdot 12=4\cdot 9$
Ted: did not record the chunk in his notes	Kazimir: What is $\mathbb{Q}$? $\mathbb{Q}=\{\frac{a}{b} \mid a,b\in\mathbb{Z},\ b\neq 0\}$ $\frac{a}{b}\leftrightarrow(a,b)\in\mathbb{Z}\times(\mathbb{Z}-\{0\})$ ★ $\frac{3}{4}=\frac{9}{12}$ but $(3,4)\neq(9,12)$
Petra: $\mathbb{Q}=\{\frac{a}{b} \mid a,b\in\mathbb{Z},\ b\neq 0\}$ $\frac{a}{b}\longleftrightarrow(a,b)\in\mathbb{Z}\times(\mathbb{Z}-\{0\})$ $\frac{3}{4}=\frac{9}{12}$ $(3,4)\neq(9,12)$	$\mathbb{Q}=\{\frac{a}{b} \mid a,b\in\mathbb{Z},\ b\neq 0\}$ $\frac{a}{b}\longleftrightarrow(a,b)\in\mathbb{Z}\times(\mathbb{Z}-\{0\})$ $\frac{3}{4}=\frac{9}{12}$, but $(3,4)\neq(9,12)$ Landon:

$\mathbb{Q}=\{\frac{a}{b} \mid a,b\in\mathbb{Z},\ b\neq 0\}$ $\frac{a}{b}\longleftrightarrow(a,b)\in\mathbb{Z}\times(\mathbb{Z}-\{0\})$

Meredith: