PREPARING STUDENTS FOR CALCULUS

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This quantitative study compared the implementation of a problem-based curriculum in precalculus and a modular-style implementation of traditional curriculum in precalculus to the historical instructional methods at a western Tier 2 public university. The goal of the study was to determine if either alternative approach improved student performance in precalculus and better prepared students for success in a calculus sequence. The study used quantitative data collection and analysis. Results indicate students who experienced the problem-based curriculum should be better prepared to learn calculus but mixed results in terms of retention and success in calculus.

Key words: Precalculus, calculus, problem-based learning

If Calculus is the gateway to higher-level mathematics, then Precalculus is the course that should prepare students to be students of calculus. Students in first-semester mathematics courses continue to receive passing grades at low rates. In a report on factors effecting student success in first-year courses in business, mathematics, and science at a western Tier 2 public university, Benford and Gess-Newsome (2006) identify student academic under-preparedness and ineffective and inequitable instructional techniques as factors that contribute to the situation. The department of mathematics and statistics has been particularly concerned about the success rate of students enrolled in Calculus. Anecdotal data indicated faculty felt students entering the calculus sequence were under-prepared. Students did not have a deep understanding of the concept of function, a "central underlying concept in calculus" (Vinner, 1992), and were not able to solve problems at the level expected in the calculus sequence. Upon examining their preparation of students for first-semester calculus, the department discovered students in Precalculus also experienced a low rate of passing grades (grades of C or higher).

Thus, as part of a university-wide initiative to improve student success in first-year courses with a high rate of non-passing grades (grades of D, F, W), the department of mathematics and statistics chose to examine two alternatives to the traditional curriculum in precalculus. The goals of this initiative were to increase the rate of passing grades in precalculus *and* calculus and improve retention rates for students in higher-level mathematics. Historically, students participating in a precalculus course experience lecture-based instruction, using a traditional textbook, with little opportunity to practice problems and engage with the content during class. In light of the report and faculty concerns, the department chose two alternative methods for teaching precalculus that focused on offering students greater opportunity to master the precalculus content, gain a deeper understanding of the concept of function, and improve their problem-solving skills.

For the first option, the department adapted a modular approach used at the University of Texas in El Paso. In this model, the precalculus curriculum is split into three time periods, Modules 1, 2 and 3. Each module is 5 weeks in length. Students must pass an exam at the end of each module to continue to the next. If a student does not pass the exam at the end of a module, they may retake the current module over the next 5 weeks. If a student does not finish all three modules by the end of the 16-week semester, they may continue the sequence the following semester (including summer semesters). The advantage of this approach is that students are able to repeat material they have not mastered without the fear of earning a non-passing grade at the end of a traditional 16-week semester. That is, this approach gives students more time to remediate, if needed. The disadvantages are (1) instruction is not changed (i.e., students continue to experience traditional, lecture-based instruction) and (2) students must pay for an additional semester of precalculus if they are not able to finish all three modules in a single semester.

The second option offered by the department was a reform-based curriculum focused on a quantitative approach to learning concepts in precalculus (need to look up this reference) and a problem-based classroom environment. This curriculum was specifically designed to develop students' conceptual understanding of function (including trigonometric functions), problem solving abilities and skills that are foundational to calculus. Students engaged in problem-based learning in groups on a daily basis. Lecture became the exception, rather than the rule, and students were expected to learn mathematics through investigating problem situations. The advantages to this curriculum are students (1) engage in solving problems every class period; (2) learn by "doing mathematics," and (3) use a research-based curriculum that reflects what students need to know to achieve success in calculus. The disadvantage to this curriculum is that instructors and students are often unfamiliar with teaching and learning in a problem-based environment using group learning. Thus, establishing classroom norms may take longer than in a traditional college course.

The research questions for this study were as follows:

1. Does implementation of a problem-based curriculum or the adaptation of the modular approach improve student success in *Precalculus Mathematics* compared to traditional instructional methods?
2. Does implementation of a problem-based curriculum or the adaptation of the modular approach improve student preparation for *Calculus I* compared to traditional instructional methods?

Theoretical Framework.

The theoretical framework for this study combined ideas from work on the reasoning abilities and understandings students need to be successful in calculus (e.g., Selden & Selden, 1999, Jensen, 2010), Social Cognitive Theory (Bandura, 2001), and research on the relationship between students' attitudes toward mathematics and mathematical achievement (e.g., Alkhateeb & Mji, 2005). It is well documented that a complete notion of function, covariation, function composition, function inverse, quantity, exponential growth, and trigonometry are essential to learning in precalculus and calculus (Dubinsky & Harel, 1992; Rasmussen, 2000; Carlson et al., 2002; Engelke, Oehrtman & Carlson, 2005; Oehrtman, Carlson & Thompson, 2008; Carlson, Oehrtman & Engelke, 2010). In addition, Stanley (2002) found that students who experience problem-based learning in precalculus increased their ability to solve real world problems, identify and use appropriate resources, and take a more active role in their learning. Using these results, the research team chose a research-based curriculum for experimental group A that included a problem-based approach to learning and emphasized development of the function concept, covariational reasoning, and trigonometry. These results also informed the selection of the tool used to assess student preparation for calculus (see Methodology).

Social Cognitive Theory (SCT) holds that human behavior is often predicted by what students believe they are capable of rather than the realization of their capabilities (Bandura, 2001). In other words, students determine what to do with specific mathematical knowledge and skills by their self-efficacy rather than what they might actually understand mathematically. Their behavior is part of a three-way reciprocal interaction between personal factors (e.g., cognition and affect), behavior and the environment. The design of this study assumed that a student's affect about mathematics will impact their desire to continue their mathematical learning and success in the subsequent calculus sequence, an assumption supported by several studies (Lester, Garofalo, & Kroll, 1989; House, 1995; Randhawa, Beamer & Lundberig, 1993). Hence, assessment of student success included a survey of student efficacy around learning in mathematics.

Methodology.

This project used a quantitative approach of program evaluation across three types of course offerings available at a western Tier 2 public university during the 2010/2011 and 2011/2012 academic years. Quantitative methods were used to measure student preparation for first semester calculus and retention in precalculus and calculus. In addition, qualitative methods were used to describe differences in instructor teaching strategies that might interact with the data collected through quantitative methods. This inclusion of qualitative description helped the investigators identify any mediating variables attributed to instructional styles.

To answer research question 1, we measured overall student success in Precalculus using end-of-semester grades. To answer research question 2, we analyzed scores from the Precalculus Concept Assessment Tool (PCA; Carlson, Oehrtman & Engelke, 2010) and pass/fail rates among students who completed Calculus I the semester following completion of Precalculus. The 25-item PCA multiple-choice test is a valid and reliable instrument that measures "the reasoning abilities and understandings central to precalculus and foundational for beginning calculus." Eighteen items assess student understanding of the concept of function; five items assess student understanding of trigonometric functions; and four items assess student understanding of exponential functions. In addition, ten items require students to solve novel problem situations using quantitative reasoning and ideas of function, function composition, or function inverse. However, we recognize that instructional methods in Calculus I at this particular university might not align with research-based instructional practices in teaching and learning Calculus. Hence, we also compared student grades in Calculus I among students who completed the course the semester immediately following completion of Precalculus.

All students enrolled in Precalculus were required to complete the PCA instrument. However students were able to choose whether their PCA score was included in the study, and students' class grades were not based on their performance on the PCA. In the control group (traditional curriculum, primarily lecture-based instruction) and the experimental group A (the reform-based curriculum), the PCA was administered during the last week of classes for each semester. In experimental group B (the modular approach using a traditional curriculum), the PCA was administered during the last week of Module Three. Student efficacy around learning in mathematics was measured through the Mathematics Confidence and Attitude Survey (Piper, 2008). This survey was administered via email using the Google Education Suite in the final week of each term.

Results.

In order to determine if a problem-based curriculum or the adaptation of the modular approach improved student success in Precalculus compared to traditional instructional methods offered at this university, we compared end-of-semester grades for the 2010/2011 and 2011/2012 academic years using a *t*-test with the type of curriculum (tradition, modular or problem-based) used as the independent variable. At this university, student success is defined as completing a course with a letter grade of A, B or C. A letter grade of D or F is considered failure since it does not earn a student credit toward their degree. Hence, we compared the mean pass/fail rate for each type of curriculum. Over these two academic years, descriptive statistics indicate that students who experienced the modular approach or the problem-based curriculum were more successful in Precalculus, with students experiencing the modular approach enjoying slightly higher success rates.

	Curriculum	N	Mean	Std. Deviation	Std. Error Mean
Pass/Fail	Traditional	882	.71	.452	.015
	Modular	804	.83	.372	.013
	Problem-based	376	.79	.406	.021

Table 1. Mean pass rates for 2010/2011 and 2011/2012 academic years

The differences in mean pass/fail rates were statistically significant between the traditional and modular approach and between the traditional and problem-based approach with *p*-values of .000 and .004, respectively. There was not a statistically significant difference between the mean pass/fail rates for the modular approach and problem-based curriculum.

Curriculum	**Difference**	**Standard Error**	***t*-ratio**	***p*-value**
Traditional/Modular	-0.120	0.20	-5.935	.000
Traditional/Problem-based	-0.078	0.027	-.2.896	.004
Modular/Problem-based	0.042	0.024	1.756	.079

Table 2. t-test statistics for Mean Pass rate

Student scores on the PCA from the 2010/2011 and 2011/2012 academic years were compared using a *t*-test with the type of curriculum (tradition, modular or problem-based) used as the independent variable.

	Curriculum	N	Mean	Std. Deviation	Std. Error Mean
Pass/Fail	Traditional	311	7.91	3.289	.187
	Modular	96	8.17	3.361	.343
	Problem-based	111	10.78	3.502	.332

Table 3. PCA mean scores for the 2010/2011 academic year

	Curriculum	N	Mean	Std. Deviation	Std. Error Mean
Pass/Fail	Traditional	69	7.19	3.112	.375
	Modular	347	8.76	3.505	.188
	Problem-based	171	10.17	4.219	.323

Table 4. PCA mean scores for the 2011/2012 academic year

It should be noted that this university transitioned out of the traditional, lecture-based curriculum after the Fall 2011 semester. Only the modular approach and the problem-based approach were offered in the Spring 2012 semester. Thus the n=69 for the traditional curriculum is much lower than one might expect. This was accounted for in subsequent t-tests for independent samples by using a t-test for unequal variances between the traditional curriculum and the modular approach.

Curriculum	**Difference**	**Standard Error**	***t*-ratio**	***p*-value**
Traditional/Modular	-1.567	.419	-3.737	.000
Traditional/Problem-based	-2.981	.561	-5.312	.000
Modular/Problem-based	-1.415	.351	-4.032	.000

Table 5. t-test statistics for mean PCA scores 2010/2011 and 2011/2012

Descriptive statistics show that the mean score of students who experienced the problem-based curriculum was greater than the mean score of students who experienced the traditional curriculum or the modular approach in both academic years. Furthermore, the difference in mean scores was statistically significant between all three curricula with *p*-values less than 0.0001.

Student semester grades in Calculus I were compared for students who completed Calculus I the immediate semester after completing Precalculus. Grades were taken from the Spring 2011, Fall 2011, Spring 2012, and Fall 2012 semesters. We were only interested in whether experiencing a specific curriculum in Precalculus helped students pass Calculus I. Hence, we analyzed semester grades in terms of passing score (i.e., A, B or C) and failing scores (i.e., D or F). Scores were analyzed across the population of students satisfying the above requirement. We used independent sample t-tests to compare the pass/fail rate in Calculus I between students who experienced each type of curriculum in Precalculus.

	Curriculum*	N	Mean	Std. Deviation	Std. Error Mean
Pass/Fail	Traditional	169	.61	.489	.038
	Modular	205	.69	.465	.032
	Problem-based	62	.65	.482	.061

Table 6. Mean pass rates for Calculus

Descriptive statistics show that the mean pass/fail rate in Calculus I for students who experienced the modular and problem-based curriculum were slightly higher than the pass/fail rate for students who experienced the traditional, lecture-based curriculum in Precalculus. However, the differences are not statistically significant at the $\alpha = 0.05$ level.

Curriculum	**Difference**	**Standard Error**	***t*-ratio**	***p*-value**
Traditional/Modular	-0.078	0.50	-1.576	.116
Traditional/Problem-based	-0.036	0.072	-.496	.621
Modular/Problem-based	0.043	0.069	.615	.540

Table 7. t-test statistics for mean pass rates in Calculus

Since the population sizes were so different for the control group and both experimental groups, we also took a simple random sample of 60 scores from each population (i.e., students who completed the traditional curriculum in Precalculus, the modular approach, or the reform-based curriculum) to verify the results above.

	Curriculum	N	Mean	Std. Deviation	Std. Error Mean
Pass/Fail	1	60	.57	.500	.065
	2	60	.70	.462	.060
	Problem-based	60	.63	.486	.063

Table 8. Mean pass rates for Calculus with simple random sample

Curriculum	**Difference**	**Standard Error**	***t*-ratio**	***p*-value**
Traditional/Modular	-0.133	0.088	-1.517	.132
Traditional/Problem-based	-0.067	0.090	-0.741	.460
Modular/Problem-based	0.067	0.087	0.770	.443

Table 9. t-test statistics for mean pass rates in Calculus with simple random sample

Descriptive and t-test statistics for the simple random sample of 60 students in each group show similar results. The mean pass/fail rate in Calculus I for students who experienced the modular and problem-based curriculum were slightly higher than the pass/fail rate for students who experienced the traditional, lecture-based curriculum in Precalculus. However, the differences are not statistically significant at the $\alpha = 0.05$ level.

Connection to Theory and Practice.

Precalculus and Calculus I are staples of the curriculum of STEM degrees across the country. For many students, these courses are hurdles or barriers that delay or impede their degree progress. Furthermore, Calculus I instructors may often be disappointed in their students' knowledge of precalculus concepts. While many colleges and universities deliver these courses in the traditional lecture format, others are experimenting with other methods, including problem-based and modular curricula. In theory, curricular decisions should be based on which curriculum is most likely to promote student success. In practice, other factors are also part of the curriculum decision-making process, such as the availability of financial, human, and physical resources that are needed to implement the curriculum.

The popularity of the traditional lecture format may be historical, but it probably requires the least resources. Generally, all that is needed is a chalkboard and a piece of chalk, or a PowerPoint presentation and a projector. On the other hand, modular-based curriculum can be logistically more difficult to schedule and staff. In addition, faculty probably need additional time to prepare for classes that use a problem-based curriculum than those that use a lecture format. How to balance providing the most effective curriculum and pedagogy with the reality of available resources will continue to be an issue that colleges and universities must face.

At our university, we have moved from a traditional lecture-based format to a modular-based curriculum. It is uncertain whether this is a permanent change—only time will tell. What we do believe is that, for us, the traditional lecture format is the least effective of the three formats discussed here. This is supported by the data presented above that suggest that students from precalculus sections taught in the traditional lecture format are not as successful as those taught in the modular or problem-based format as measured by their precalculus grade or subsequent success in calculus.

The results of this study contribute to the knowledge base of best practices that are associated with the teaching and learning of precalculus and calculus. Although further research is needed, these results suggest that the traditional lecture format found in most university and college classrooms may not be the most effective method of instruction. Rather, students may learn best by being exposed to problem-based curricula that allow them to explore mathematical content in a way that develops their conceptual understanding of the mathematics instead of only their algorithmic knowledge of the procedures. We hope that these results will prompt precalculus teachers (including those at our own university) to reexamine their instructional strategies and practices.

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