Students' Sense-Making in Mathematics Lectures

Aaron Weinberg Emilie Wiesner Timothy Fukawa-Connelly
Ithaca College Ithaca College University of New Hampshire

It is important for us to develop an understanding of how students make sense of lectures, particularly in upper-level mathematics classes, where the focus is on definitions, proofs and examples. To do this, we observed abstract algebra lectures, collected students' notes, and interviewed students about their experiences while re-reading their notes and re-watching the lectures. Our preliminary findings indicate that students attempt to make sense of components of the lecture on multiple levels, which influences their understanding of the mathematical aspects of the lecture.

Key Words: Lectures, Abstract Algebra

Despite efforts to make college classrooms more inquiry-based, typical college students still spend approximately 80% of their time in class listening to lectures (Armbruster, 2000). Lecture listening is a difficult cognitive task for college students (Ryan, 2001), but it is important, since what students take away from lectures is closely linked to what they learn (e.g., Titsworth & Kiewra, 1998).

The goal of this research project is to develop and use a framework to describe the ways students make sense of mathematics lectures and the various factors that influence and constrain this sense-making process. In the work reported here, we focus on the research questions:

1. In what ways do students make sense of upper-level mathematics lectures?
2. What aspects of the lecture influence the ways students make sense?
3. What aspects of the student—their beliefs, habits, and knowledge—influence the ways they make sense of lectures?

Background

Most prior research on students' participation in lectures has focused on their note-taking habits. Students' notes can be viewed as a "symbolic mediator between the content taught by the teacher and the knowledge constructed by students" (Castello & Monero, 2005, p. 268), and Ryan (2001) generated metaphors that students might use to describe and guide their note-taking practices and lecture-observing habits. However, this—and much of the prior research (e.g. Van Meter, Yokoi, & Pressley, 1994)—has focused solely on students' self-reported habits rather than empirical evidence and did not describe the ways that these various habits might affect students' learning. Most other research has focused on factual recording and recall rather than deeper understanding of the content (e.g. Kiewra, 1991; King, 1992). In addition, there has been little previous research that has described the ways that students' engage in *mathematics* lectures.

Initial Framework

Although one of the goals of this research is to construct a framework for analyzing data, we have adopted an initial theoretical lens to help us identify aspects of the lecture or of students' beliefs and habits that might influence the ways the make sense of the lecture.

Sense-Making

The terms "sense" and "sense-making" are widely used in mathematics education, appearing in research literature (e.g., Schoenfeld, 1992), the Common Core Standards (National

Governor's Association, 2010), and an NCTM series (e.g. NCTM, 2011). Although these terms do not have standard definitions, they are typically used in conjunction with describing the act of constructing a personal understanding of a mathematical fact, procedure, concept, or theory.

Lecture Components

Lectures contain numerous components that students must attend to and interpret including proofs, definitions, statements of theorems and algorithms. These components can be broadly distinguished by their mode of presentation: written, spoken, and gestural. Beyond this, we divide lecture components into communicational aspects and mathematical aspects. *Communicational aspects* of a lecture include immediacy, gesture, temporal-spatial components, and organizational cues; these cues can organize transitions from moment-to-moment, from day-to-day, and from unit-to-unit or class-to-class. The *mathematical aspects* of a lecture consist of facts, procedures/algorithms, and processes (including problem-solving, communication, justification, and representation).

Meta-Components of Lectures

A lecture may contain—or be influenced by—aspects that appear as components yet are part of a broader idea. We identify two such aspects: broad mathematical concepts and motivation, and pedagogical actions and motivation. *Mathematical concepts and motivation* are the broad mathematical ideas that a lecture addresses (often indirectly) and the reasons for those ideas being seen as important. For example, while the components of a lecture may include specific examples of equivalence relations, understanding the general concept, its significance in mathematics, and the reason for including it in the lecture, may affect the students' sense-making. In addition to the actions that an instructor performs during the lecture, the instructor's *pedagogical motivation* is their motivation for choosing and ordering specific ways of presenting their ideas. For example, a lecturer might present specific examples before stating a general rule in order to illustrate that rule. Understanding this pedagogical motivation may affect the way a student makes sense of the components of the lecture.

Data Collection and Analytical Methods

We have collected and are in the process of analyzing data in a pilot study. The participants are six mathematics majors who were enrolled in a standard abstract algebra class. The class was videotaped 6 times over the course of the semester. The video of the observations was transcribed to describe the written, spoken, and gestural components. In this way, the class observations were designed to capture as much of the "text" of the lecture as possible. After each observed class period, we collected the participants' notes. We identified places in the students' notes that didn't match what the instructor wrote or said.

We interviewed the students after each recorded lecture. We showed the students video clips of the lecture along with their notes, and asked questions designed to explore how they approached the class, how they took notes, how they understood the purposes of notes, and questions about the mathematical content of the class. The interview data was analyzed using grounded theory (Strauss & Corbin, 1994) through an iterative process of identifying themes, creating codes, applying the codes to the interviews, and comparing our results with each other.

Results and Analysis

We are in the preliminary stages of using this coding manual to fully describe students' sense-making. However, we have so far identified some common themes (supporting excerpts will be shown during the full presentation).

Of the six students we interviewed, five indicated that they try to copy down what the instructor writes on the board. The claim of “copy what’s on the board” seems to imply that students *aren’t* trying to understand the lecture. However, as they looked back at their notes and re-watched sections of the lecture, the students described numerous acts of sense-making. In addition, students’ described their note-taking habits as mediated by their goals, including using their notes for later reference and completing homework and exams.

Although most of the students tended to copy what the lecturer wrote on the board, there were places in every student’s notes that differed from what was on the board. Many of these discrepancies occurred at the same place where the student was able to clearly and accurately describe both the lecture component and various meta-components. For example, several students initially struggled to describe the connection that the lecturer described between ordered pairs and rational numbers. Although they could articulate an understanding of a specific component (e.g., the equivalence of two fractions), they couldn’t articulate either the meta-component of the role of equivalence relations or the instructor’s pedagogical motivation for bringing up the specific numerical example. However, at a later point in the lecture one student added to her notes: “Each [ordered pair] should be thought of as, in this case, rational numbers.” When she got to this point while re-watching the lecture during the interview, she was able to articulate the connection.

From the interviews, it appeared that students were trying to make sense of the lecture as they observed it. However, their sense making occurred on multiple levels. Some students were able to make sense of individual mathematical components of the lecture—such as the equivalence of two fractions. Other students were able to make sense of the communicational aspects of the lecture and use the instructor’s organization (e.g. of the board space) to develop an understanding of how various ideas are related. Still other students attempted to make sense of the meta-components of the lecture, using this understanding of the significance of various mathematical concepts (e.g., equivalence relations) or the instructor’s motivation (e.g., presenting several examples before a general case) to make connections between ideas. In particular, students who *didn’t* make sense of these additional components struggled to describe the mathematical ideas in the lecture.

The students all described various barriers to making sense of the lecture. Some students had difficulty translating the instructor’s speech into written notes. Other students had difficulty encoding the layout of the notes on the board, and others described their difficulty following the connections the instructor made between the current lecture and ideas that the class had previously encountered. In addition to struggling to encode non-written lecture components in their notes, students described having difficulty figuring out what aspects of the lecture were most important.

Discussion

Despite claiming that they are simply duplicating what the instructor writes on the board, it appears that students are constantly trying to make sense of the lecture. Although they often try to copy the instructor’s writing into their notes, places where their notes deviate from what is on the board seem to signify instances where students were successful in making personal sense of the lecture components and, at times, meta-components as well.

This sense-making process is mediated by the students’ goals for taking notes and their perceptions of how they will be using the notes. The students face numerous challenges in determining what aspects of the lecture are most important; they also struggle to make sense of aspects of the lecture that are not written on the board, such as the instructor's speech, the board layout, or concepts that are connected across multiple lectures.

More importantly, students can make sense of aspects of the lecture on multiple levels. A student can make sense of a particular mathematical lecture component, an organizational cue, or a meta-component. However, if they only make sense of one of these (such as understanding that ¾=9/12, but not why this example is pedagogically relevant), then they might fail to understand some of the most important aspects of the lecture.

Discussion Questions

1. What are important aspects of students' sense-making during lectures?
2. How might various aspects of lectures affect students' sense-making habits?
3. In what ways can the initial framework described here be enhanced to better capture these important aspects?

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