ASSESSING STUDENT PRESENTATIONS IN AN INQUIRY-BASED LEARNING COURSE

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Inquiry-Based Learning (IBL) is an instruction method that puts the student as the focal point of the learning experience. An integral part of many IBL proof-based courses is presentation of proofs given by students. In this report, an assessment rubric used to evaluate student presentations of theoretical exercises is introduced. The W.I.P.E. rubric is based on several different assessment models, which emphasize proof writing and comprehension. The rubric was created to evaluate in-class presentations in undergraduate Abstract Algebra courses for math and math education majors, which offer graduate credit.

Key words: [Inquiry-Based Learning, Student Presentation, Assessment Rubric, Proof]

Related Literature

Inquiry-Based Learning (IBL) is a student-centered teaching method. As Schinck (2011) examined several studies on IBL, in particular Smith (2005), she noted that classroom communities of inquiry encourage students to produce proofs by making global or intuitive observations about the mathematical concepts and transform these observations into formal, deductive reasoning. The Center for Inspired Teaching released a publication containing a list of various studies that demonstrate positive outcomes associated to IBL (Sweetland, 2008). Particularly, a study involving over 1400 students showed that inquiry-based approaches in middle and high school language arts classrooms allow both low and high-achieving students to make academic gains (Applebee, Langer, Nystrand, & Gamoran, 2003). This study, along with several other studies, demonstrates the effectiveness of IBL on communities of students who have different achievement levels and background skills (Kahle, Meece, & Scantlebury, 2000).

Many students struggle with learning to become mathematical proof writers (Selden & Selden, 2003; Tall, 1991; Solomon, 2006). David Tall remarks that "students meeting new formal mathematical ideas for the first time may face difficulties in cognitive reconstruction to adapt to the new way of thinking and may need help in this transition phase" (1991). Although students struggle with formalizing mathematical ideas, mathematicians and mathematics educators both agree that having students construct mathematical proof is not only essential in teaching students to communicate and explain mathematics, but also aids students in learning how to become better problem solvers (de Villiers, 1990; Larsen & Zandieh, 2008; Hannah 1980, Hanna & Barebeau, 2008). As IBL classrooms focus on mathematical discovery and problem solving, an IBL class is an ideal setting for students to learn to become mathematical proof writers. As Tall (1999) asserts about university students "If they are given opportunities to develop mathematical thinking processes, albeit with initially easier mathematics, they may develop attitudes to mathematics more in line with those preferred by mathematicians". In a typical undergraduate proof-based IBL class, that utilizes a modified Moore method, students construct arguments and proofs of theoretical exercises from a presentation script. In many cases, the instructor will choose to use non-standard IBL textbooks, such as, Burger & Starbird (2005), Hale (2003), Schumacher (1995). During most of the class time, students present proofs of problems from a presentation script.

In this study, a rubric used to evaluate student presentations of theoretical exercises from the presentation script is introduced. This rubric is based on several well-known proof writing and comprehension assessment models, including models given by de Villerie's (1990), Andrew (2009) and Yang & Lin (2008). This rubric has been used in undergraduate

Abstract Algebra courses for math and math education majors that offer graduate credit. In these courses, class meets two days per week for seventy-five minutes. One day a week students present material and on the other day there is lecture-based instruction. After their presentations, students turn in written proofs of the material they presented. The next class students are given back their written submission of their proof along with a graded rubric that contains comments about their presentation.

The W.I.P.E. Rubric

To aid in the efficiency and uniformity of evaluating student presentations of theoretical exercises, the W.I.P.E rubric was created. This rubric considers four categories involved with presentations of proofs: **W**ritten proof, **I**nteraction with peers, **P**roof functions, and **E**xplanation of proof. Each category has equal weight. Below is an explanation of each category and the way in which presentations are evaluated related to each category.

Written: In this category the student's written proof, which is typically completed before class, is assessed. Evaluation of the written proof is primarily based on the assessment model of Andrew (2009). Andrew's PEET evaluation system allows for efficiency and easy identification of errors related to structure and understanding of the proof. Particularly, related to the PEET system, the types of structural errors that are examined are errors in logical ordering of ideas and proof readability. The errors of understanding that are assessed are not enough justification for arguments, aspects of the proof not addressed and forgotten conclusions. If a student receives a lot of feedback from their peers during their presentation, the presenter is allowed to rewrite their write-up and turn in at the end of the day. Thus, the student benefits from peer and self-assessment.

Interaction: After each presentation, students open the floor to their peers for discussion. During this time, presenters are given the chance to address any questions or refutations made by the class. In addition, the presenter can further justify and argue the statements made in his/her proof. Typically, the group is careful in discussing and reviewing each proof for validity, as they know they are responsible for this material. In Proofs and Refutations Latkos (1976) outlines methods of mathematical discovery. Larsen and Zandieh (2008) restructured these methods of mathematical discovery from a pedagogical perspective. Below is a table from Larsen's and Zandieh's work summarizing their framework.

Type of activity	Focus of activity	Outcome of activity
Monster-barring	Counterexample & underlying definitions	Modification or clarification of an underlying definition
Exception-barring	Counterexample & conjecture	Modification of the conjecture
Proof-analysis	The proof, the counterexample, & the conjecture	Modification of the conjecture & sometimes a definition for a new proof-generated concept

How well the student appropriately addresses the concerns raised by their peers is the primary focus of this assessment category. Students should engage in argumentation in a logical, sensible way, i.e. an argument that follows Toumlin's (2003) model. In the assessment of the students' interaction, related to the table above, the instructor has to determine what type of activity fits the refutation brought by the class. Then the instructor has to assess if the presenter responded with the appropriate activity outcome.

As the presenter's validation is usually the only source of truth for an argument, issues of persuasion versus conviction detailed in Inglis and Mejia-Ramos' (2009) work commonly arise during this interaction. In class, students naturally pick other peers who they view as mathematical authorities. The class assumes arguments made by the student who is viewed as a mathematical authority are valid without dispute. This usually leads to limited interaction between the class and this presenter.

Proof: As given by Bell (1976) and then expanded by de Villeriers (1990), the functions of mathematical proof are: Verification, Explanation, Systemization, Discovery, and Communication. Evaluated in this category is how well the students address each of the five functions. The overall validity of the proof is evaluated in this category. The assessment in this category focuses on different aspects than the evaluation of the written proof since the evaluation of the written proof considers only structure and understanding. Also, the presenter can augment and adjust the written proof after receiving feedback from peers, which is not possible for the proof presented in-class.

Explanation: The main assessment factor that is evaluated here is how well the presenter displays their comprehension and understanding of the proof being presented through their explanation. Most of the evaluation process of this category has foundation in Yang and Li's Reading Comprehension of Geometry Proof (RCGP) model (Yang & Li, 2008), which includes four levels and five facets of proof comprehension. Specifically, the presenter's explanation should demonstrate their understanding of basic knowledge required to prove the given statement. Then the student should be able to explain how they logically chain arguments and give a summary of the proof. If suitable, explanation about generalizing and applying ideas found in the proof should be given. Recently, Mejia-Ramos, Fuller, Weber, Rhoads, Samkoff, (2012) have extended the Yang and Lin's model to further include additional local and holistic aspects of proof comprehension assessment, which are now being examined in relation to the W.I.P.E. rubric.

Future Research

In the W.I.P.E. rubric, the current categories are chosen to evaluate three primary aspects: student comprehension, mathematical communication and problem-solving skills. This rubric was created and refined after evaluating student presentations every semester since fall of 2010. Out of the courses included in this study, there was an average of 7-10 students, which allowed each student to present 5-8 proofs per semester. In the future, I hope to work with others who teach proof-based courses to use the W.I.P.E. rubric to evaluate student presentations. This will help to strengthen the rubric as there maybe categories that need to be adjusted or changed to suit instructor's needs. I also plan to adjust the W.I.P.E. rubric to assess student presentation in non-proof based courses. The recent work of Mill's (2010) investigates what mathematics faculty contemplate as they plan lectures that include proof presentations. Future research will incorporate the findings of Mill's work to help refine the W.I.P.E. rubric to make it a more effective assessment tool.

Questions:

1. In each of the assessment categories of the W.I.P.E rubric, are there certain aspects of assessment that should be considered that were not mentioned?
2. Should one of the evaluation categories be given more weight than another?
3. Are there other assessment models that can be utilized to help strengthen any of the evaluation categories of the W.I.P.E. rubric?

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