HOW PRE-SERVICE TEACHERS IN CONTENT COURSES REVISE THEIR MATHEMATICAL COMMUNICATION

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Math content courses aim to develop mathematical reasoning and communication skills in future teachers. Instructors often assign problems requiring in-depth written explanations to develop these skills. However, when a student's conception is incorrect, does written feedback from the instructor create the cognitive dissonance necessary to effect realignment of the student's understanding? These conceptions may be mathematical ("what is a fraction?") or meta-mathematical ("what constitutes a justification?"). Assigning problem revisions theoretically creates space for cognitive dissonance by having students rethink their solutions. I investigate a revision assignment in a course for future teachers to understand the nature of students' revisions and the possible impetuses for these revisions. In particular, I find preliminary evidence that students' revisions demonstrate changes in their language, mathematics, and use of examples and representations. Further, students' adoption of new representations in their solutions are largely due to observing peers' presentations rather than to instructor feedback.

Key words: **Pre-service elementary teachers, mathematical communication, mathematical justification, revision, representations, inquiry-based learning**

Research Questions

One goal of math content courses for future teachers is to develop mathematical reasoning and communication skills. However, unlike courses for math majors, future teachers need to be able to reason with conceptual and visual models and not just the axiomatic reasoning of mathematicians. As in most other math courses, instructors often assign problem sets requiring in-depth written explanations to develop and assess these skills. However, when a student communicates a concept incorrectly, does the standard written feedback from the instructor create the cognitive dissonance to effect a realignment of the student's understanding? The conceptions I refer to may be mathematical ("what is a fraction?") or meta-mathematical ("what constitutes a justification?"). Assigning problem set revisions could theoretically create a space for cognitive dissonance, if not the dissonance itself, by asking students to rethink their solutions. I investigate a revision assignment in a course for future teachers to better understand the nature of students' revisions and the possible impetuses for these revisions.

I had several questions about this revision process:

- Q1 What kinds of revisions do students make when asked to revise their solutions to problem sets?
- Q2 What differences can be detected in the influence of peers' in-class presentations and instructor feedback in the revision process?

These questions are important for several reasons. In particular, they address specific practices of teaching mathematical justification and communication. The first question assesses the usefulness of the revision activity as a method of improving students' mathematical justification and communication. The second question illuminates the potential importance of multiple forms of feedback in math content courses for future teachers.

Literature

It is widely acknowledged that teachers need strong mathematical reasoning and justification skills to successfully build mathematical concepts in the classroom (Ball & Bass, 2003; Ball, 1993). Further, there is evidence to suggest many preservice teachers are weak in these skills (Ball, 1990; Morris, 2007). Unfortunately, there is a lack of literature on how preservice teachers can gain these skills (Hiebert & Morris, 2012).

The tools teachers should use in justification are more than the axiomatic systems of mathematicians. They should use conceptual models (Lamon, 1997) and visual representations of those conceptual models (NCTM, 2000). Examples of these include: the number line, the array model of multiplication, and the chip model of integer arithmetic. Given the needs of their future teaching practice, mathematical solutions written by future teachers should utilize these models and representations.

Another area this project addresses is revision of mathematical writing. While there is an ample body of literature on the research methodology for revision of written composition, their heavily linguistic methods do not lend themselves to analyzing mathematical revisions (Fitzgerald, 1987). In a mathematical revision, it is not just the linguistic changes we care about, but the changes in validity of mathematical arguments, clarity of mathematical ideas, and use of mathematical tools (e.g. algebraic models, diagrams, examples). Part of this study seeks to fill this hole by using empirical examples of student revisions to create a systematic framework to describe revisions to written mathematical problem sets (see Q1).

Theoretical Framework

I initially looked to Balacheff's model of a mathematical *conception* as a useful framework for thinking about the pedagogical purpose of a revision activity and the role of feedback in the revision process. In Balacheff's model, a conception is a provisional state of equilibrium of an action-feedback loop between a *milieu* (i.e. learning environment, such as a classroom), and a *subject* (e.g. a student) under proscriptive constraints (Balacheff & Gaudin, 2009). In this model, an instructor's role is to create a milieu that perturbs the equilibrium and the student's job is to select features of the environment to use as feedback. Using this language, after the students create a first draft of their problem sets, the instructor introduced new sources of feedback into the milieu (e.g. student presentations and instructor feedback) and created the explicit need for students to choose from sources of feedback multiple times (e.g. multiple drafts of assignments). This theory is meant to model students' understanding surrounding a mathematical concept or problem, but is problematic when their written output is a proxy for their understanding. A written assignment is a reflection of a student's verbal communication skills, not just reasoning skills. One way to remedy this is to analyze a student's conception of what makes a good written solution, rather than his or her conception of a mathematical topic. This better represents the entwined nature of understanding and communication.

Data

In this math content sequence for pre-service teachers, students completed weekly problem sets comprising challenging problems that require reasoning, justification, and explanation. The course was run using inquiry-based learning (IBL) pedagogy and students presented solutions to problems every week in class. Often, several students present different solutions to the same problem while the class as a whole contributes clarifications, corrections, and modifications to presented solutions. During this discussion students annotated their problem sets with colored pens before turning them in to the instructor for initial feedback. I call these annotations *students' marker comments*. At three points in the semester, students selected problems to revise and resubmit.

The data collected includes the following: first drafts of problem, including students' marker comments; initial instructor feedback on first drafts; final drafts of problem sets; and written student reflections their revision process. The students' marker comments serve as a proxy to understand the effect of peer presentations and class discussion on the revision process. Figure 1 represents the revision process. The items in red are the artifacts I have access to.

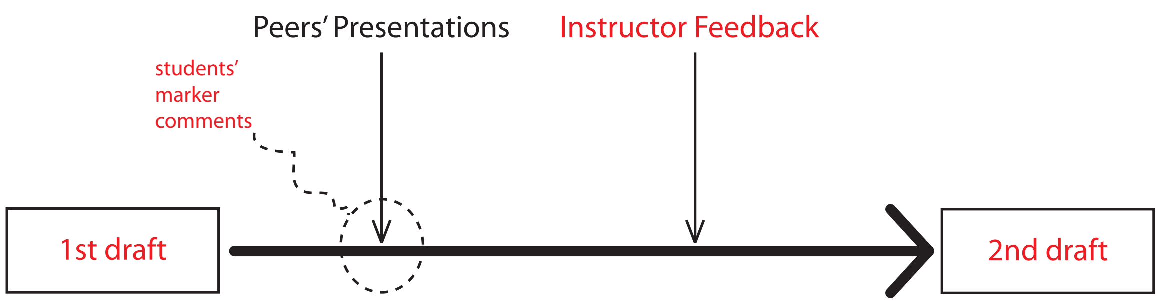

Figure 1: Model of student revision process. The items in red represent artifacts I can see.

I will use the term *one student-worth of data* to describe all of those pieces of data for one student's revision of one problem.

Methods

To understand students' revisions as well as sources for their revisions, I wanted a framework that would both describe the *changes* students made during the revision process as well as *potential changes*—students' marker revisions made during peers' presentations and instructor feedback. To develop this framework I started by analyzing 10 students-worth of data. I wrote memos describing (1) changes I detected between drafts, (2) students' marker comments on their first drafts, and (3) instructor feedback. Figure 2 shows how these memo-ed objects fit into the revision process.

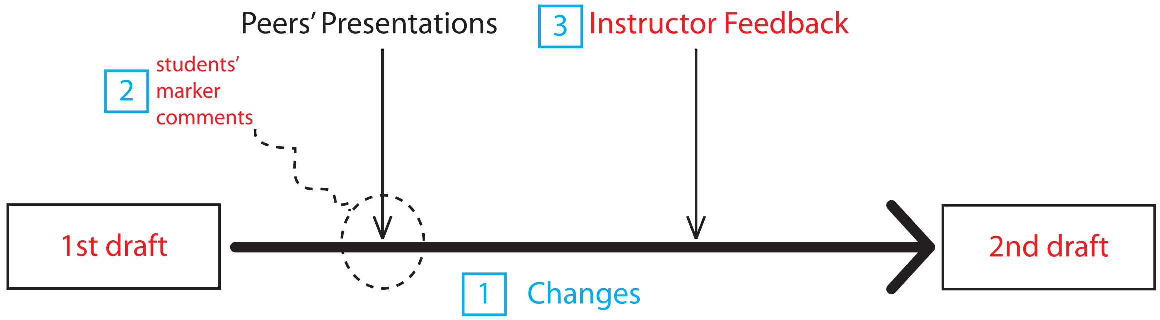

Figure 2: I wrote memos describing [1], [2], and [3] seen in this figure.

I identified themes in the memos and drafted a set of codes and their definitions. I next used this preliminary coding scheme to code the remaining 20-students worth of data from that particular problem set and revised the code definitions when ambiguities arose. Next, I continued that revision process with a new problem set, another 30 students-worth of data, until the definitions stopped changing. This second set of data is what I present below.

One subtlety of the coding is that there is a challenge of systematically identifying every single change between first and second drafts. So rather than identify every single change and count the codes with multiplicity, I used the list of codes as a checklist and recorded only if a certain code occurred or not. This proved to be more systematic and replicable when re-coding for reliability. Further, I used written student revision reflections as member checking; students had written reflections describing the changes they made and I was able to cross-check the changes I identified using their reflections.

Results

The revised codes and definitions arising from this process can be found in Table 1. The codes fall into four larger families: Language (Expo, Expl, Lang), Mathematics (Lar, Sma, Jus, Not, Prec, Def), Examples, and Representations.

Table 1: Revision Framework

Code	Name	Definition
Expo	*Adds Exposition*	This refers to *adding* prose describing the problem premise and goal before steps are carried out. Many students do not include this in early drafts, but it appears in later drafts. This is a specialized kind of explanation, but one so common it gets its own code. This does not refer to editing the exposition, only adding it where there was none before.
Expl	*Adds Explanation*	This includes explaining a diagram, explaining thinking or attempts tried, or making steps in a process explicit. This is different then justification, as it does not explain *why* a process works; it is a *description* of a process or phenomenon. This refers to *adding* explanation to a place in a solution where there was none before, not refining it.

Code	Name	Definition
Lang	*Language*	This describes changes (not content additions) to language. Examples are changes to clarity, flow, word-use, grammar, full sentences, or conciseness. This refers to essentially the same content reorganized or reworded. It does not refer to new content such as an expanded explanation or justification.
Lar	*Large Correction*	This refers to correcting large errors in the original not covered by other codes. For example, this would not refer to filling in gaps in justification or including needed definitions. Instead this would include rectifying missing solutions or large numerical errors.
Sma	*Small Correction*	This refers to correcting small, usually numerical, errors in the original solution.
Jus	*Adds Justification*	This describes added attempts at answering "why?" or filling in logical gaps.
Not	*Introduces New Notation*	This describes a change in notation. One common example is using algebraic notation when previous description was verbal.
Prec	*Adds Precision*	This includes adding constraints on variables, using more precise quantifiers, referencing hypotheses of the problem, or correcting use of the equals sign. Adopting algebraic notation only counts as **Prec** if the previous notation was imprecise.
Def	*Definition*	Adds references to definitions, refines references to definitions, or makes them more explicit.
Ex+	*Add Example*	This refers to adding new examples in the course of an exposition, explanation, or justification.
Ex-	*Delete Example*	This refers to deleting examples from the exposition, explanation, or justification.
Rep+	*Add Representation*	This refers to adding visual/physical models of numbers and/or operations. This could be adding a number line, discrete models of numbers, area models of multiplication, etc. This does not include tables or other record-keeping and problem-solving systems. It also does not cover inclusion of an algebraic representation of a situation; that falls under **Not.**
RepM	*Modify Representation*	This refers to changes made to an existing representation, such as adding lines, circles, or arrows to an existing array or number line model.
Rep-	*Delete Representations*	This refers to deleting a representation from an earlier draft.

To give examples and non-examples of every code here is beyond the scope of this preliminary report, but I will give some flavor of the student work and the codes by providing some examples of the Justification code. Consider a problem students were given: "Show that if the difference between two integers is odd, then their sum is also odd." A marker comment that received the code **Jus** was the following: "missed how to explain why it's always true that odd-even or even-odd is odd." It receive the **Jus** code because a student is noting a gap in logic that must be filled in on a later draft. An instructor comment that received the code **Jus** is the following: “Why will the two numbers always be different? (one odd and one even) Can you explain that more?”. It receive the **Jus** code because it is asking for an explanation of *why* something is true. To see an example of **Jus** coding a change, consider the following paragraph in a first draft: “If the difference between two numbers is odd, that means that one number in the pair is odd and the other is even (as seen in the examples above).” The same student in a second draft attempts (still imperfectly) to more generally address this phenomenon with the following change to her justification: “If we look at all these examples where the difference between two numbers is odd, we find out that the two numbers we are taking the difference of have to be one odd number and one even number. We know this is true for all numbers because of the rules, odd-even=odd, even-odd = odd. These two rules are the only rules that can produce an odd number as its difference.” This examples points to another subtlety in the coding—I used a code if there was a *change* in that dimension, not necessarily ultimate perfection.

When I used these codes on the revision data from one problem set (30 students worth of data), I found the results seen in Figure 3, Figure 4, Figure 5. In all three figures, a given student can receive any given code at most once, so the maximum for all codes is 30.

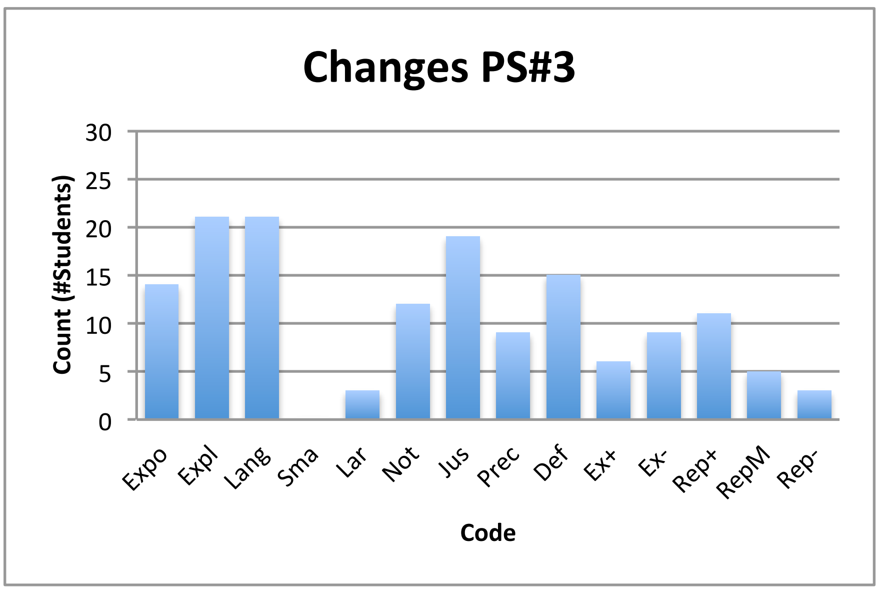

Figure 3: Changes seen between drafts in 30-students worth of data.

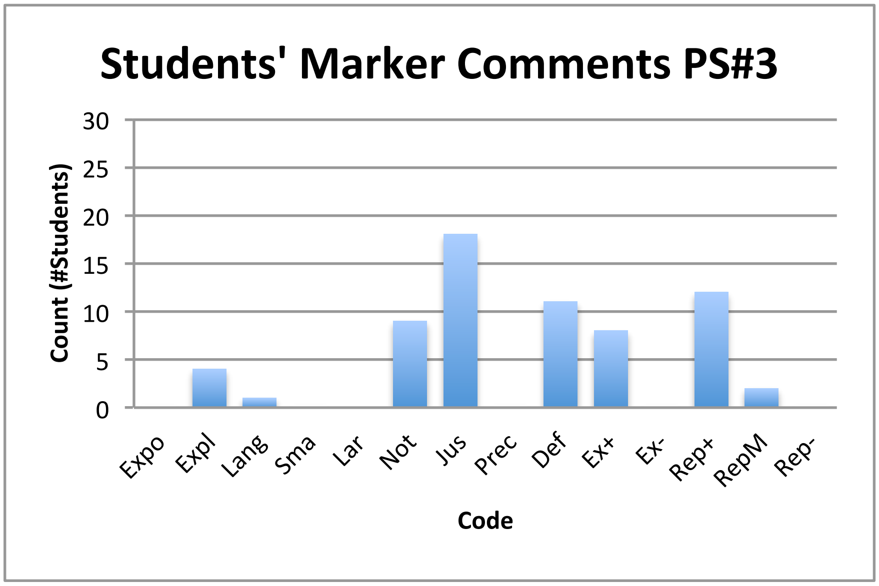

Figure 4: Potential changes seen in 30-students worth of market comments.

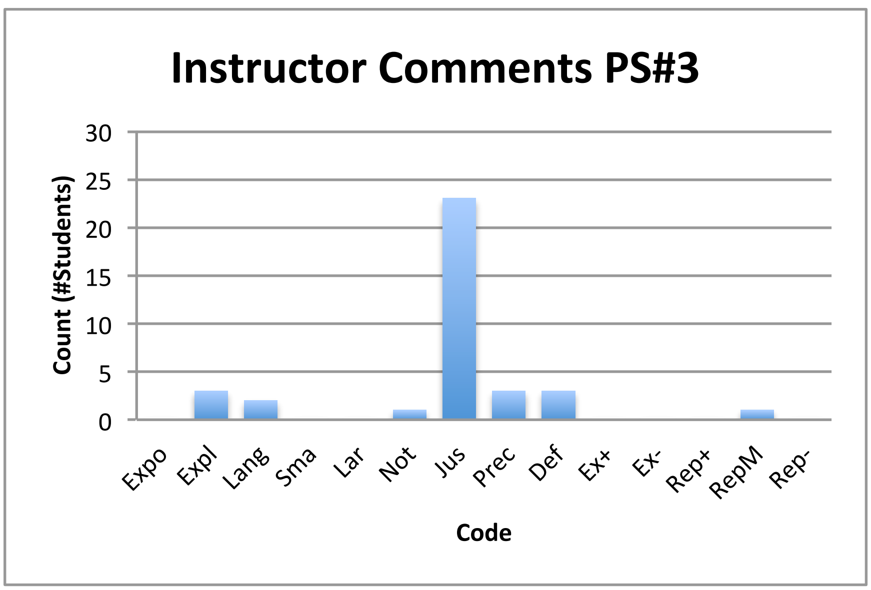

Figure 5: Potential changes seen in 30-students worth of instructor comments.

Discussion

The first observation (answer to Q1) is that students' revisions are rich mathematical changes. They make changes to their expositions, their language, their justifications, their use of definitions, their notation, their precision, their use of examples, and their use of representations. This would indicate that the revision activity is a positive pedagogical exercise.

A more nuanced observation is that peer presentations appear to affect student adoptions of examples and representations more than instructor feedback. This is in part perhaps due to the fact that (as evident in Figure 5), the instructors of this course (both mathematicians by training) fixate on formal mathematical justification in their feedback. However, despite the limited scope of this instructor feedback, over one third of the class incorporated new representations into their solutions, and almost one third of the class added or deleted examples (see Figure 3). I hypothesize that the impetus for these changes can be found in the peers' presentations, as evidenced by the students' marker comments to themselves (see Figure 4).

In addition to changes in justification, changes to examples, and changes to representations, there were changes in language (corresponding to the codes Expo, Expl, and Lang). I want to explore further what these may be due to. I hypothesize that they are both a normal part of the revision process and also highly related to the changes students made to their to justification.

I think these preliminary results have potentially interesting ramifications for using IBL methods with future teachers. We see evidence students utilize peers' presentations differently from instructor feedback, in ways that are important to their future teaching practice.

Questions to the Audience

(1) What other methodologies could I use to analyze this large collection of data?
(2) What theoretical perspectives could help me look at this data in different ways?
(3) What are some alternative frameworks for looking at (mathematical) revisions?
(4) What are some frameworks for teasing apart the code *Justification* in more detail?
(5) What is the role of the specific content area (number and operations) in this analysis? In particular, what observations could generalize to the second and third semesters of the course, whose content areas are geometry and algebra?
(6) What about these results is special to content courses for pre-service teachers? Could there be similar analyses done on student revision in upper division math courses?

References

Balacheff, N., & Gaudin, N. (2009). Modeling Students ' Conceptions : The Case of Function. *CBMS Issues in Mathematics Education*, *16*(1987), 183–211.

Ball, D. L. (1990). Prospective Elementary and Secondary Teachers' Understanding of Division. *Journal for Research in Mathematics Education*, *21*(2), 132–144.

Ball, D. L. (1993). With an Eye on the Mathematical Horizon : Dilemmas of Teaching Elementary School Mathematics. *The Elementary School Journal*, *93*(4), 373–397.

Ball, D. L., & Bass, H. (2003). Making mathematics reasonable in school. In J. Kilpatrick, W. G. Martin, & D. Schifter (Eds.), *A Research Companion to Principals and Standards for School Mathematics* (pp. 27–44). National Council of Teachers of Mathematics.

Fitzgerald, J. (1987). Research on Revision in Writing. *Review of Educational Research*, *57*(4), 481–506. doi:10.3102/00346543057004481

Hiebert, J., & Morris, A. K. (2012). Building a Knowledge Base for Teacher Education : An Experience in K – 8 Mathematics Teacher Preparation. *The Elementary School Journal*, *109*(5), 475–490.

Lamon, S. J. (1997). Mathematical Modeling and the Way the Mind Works. *Teaching and Learning Mathematical Modelling* (pp. 23–37). Albion Publishing Limited.

Mathematics, N. C. of T. of. (2000). *Principles and Standards for School Mathematics*. Reston, VA: Author.

Morris, A. K. (2007). Factors Affecting Pre-Service Teachers ' Evaluations of the Validity of Students ' Mathematical Arguments in Classroom Contexts. *Cognition and Instruction*, *25*(4), 37–41.