TRANSFER OF CRITICAL THINKING DISPOSITION FROM MATHEMATICS TO STATISTICS

Hyung Kim
The University of New Hampshire
Hwy3@wildcats.unh.edu

Tim Fukawa-Connelly
The University of New Hampshire
Tim.FC@unh.edu

In this study we draw on the constructs of eagerness, flexibility and willingness to characterize the necessary disposition for critical thinking required in learning statistics in addition to specific content knowledge (Ennis, 1989). We investigated the challenges that students who are highly successful in mathematics might have in doing statistics and found that while a student might have an inquisitive disposition and good proficiency with the foundational mathematical concepts such as functions and function transformations, that same student might struggle in statistics. Even concepts that are seemingly related to their mathematical counterparts, such as what is a variable when considering population and sample, may cause problems as the statistical sense is distinct enough from the mathematical sense. We suggest that such students may experience greater than usual affective problems in a statistics class and may, therefore, give up easier and earlier than students who have been less successful mathematically.

Keywords: Statistics, Subject-specificity, Critical thinking disposition, Transition

Recently, statisticians have collectively voiced that statistics is a discipline distinct from mathematics in several aspects (Garfield & Ben-Zvi, 2010; Wild & Pfannkuch, 1999). One difference is in the nature of the goals for problem solving. While the major goal of doing mathematics problems is to improve student understanding of the structure of mathematical content, the goal of statistics problems is gaining insights from data (Moore & Notz, 2005; Rossman, Chance & Medina, 2006). In the process of making sense of data, statisticians draw on statistical procedures determined by the context. This matter of context is another characteristic of statistics distinct from mathematics (Cobb & Moore, 1997; Rossman, Chance & Medina, 2006). Although contexts are occasionally used in mathematics problems as examples to promote student understanding of mathematical concepts, the mathematical content still has meaning free of context. In addition, the context in mathematics problems can obscure the underlying structure (Cobb & Moore, 1997). Most statistics problems, however, originate from real situations and are naturally embedded in a context, which provides meaning for numbers in statistics (Cobb & Moore, 1997). Therefore, data cannot be meaningfully analyzed without careful consideration of the given context: how the data were collected and what they represent (Garfield & Ben-Zvi, 2010). The differences in the goals and the matter of context between the two disciplines are important because they give rise to another major dissimilarity, the subject-specific critical thinking dispositions, the main focus of this paper.

According to Ennis (1989), most cognitive psychologists hold the view that critical thinking is subject specific. Under this view, different types of critical thinking dispositions are required for mathematics and statistics due to their different natures; the critical thinking disposition specific to mathematics will have to promote understanding of the structure of mathematical contents and the critical thinking disposition specific to statistics will have to promote understanding of contextual characteristics, data analysis and result interpretations. Noting the difference in subject specific critical thinking dispositions needed for mathematics and statistics, it is not uncommon to observe that students who have been successful in mathematics, both at the introductory and advanced levels, experience difficulties in translating that success to their first statistics course. This phenomenon could be attributed to the different types of critical thinking dispositions between the two disciplines; according to Ennis, simple transfer of critical thinking dispositions and abilities from one discipline to another is unlikely. Especially when students take a first statistics course after twelve years (k-12) of mathematics courses, it may take students time and effort to adjust to the new culture of statistics after the familiar culture of mathematics. Taking up this issue, in the context of the differences in subject-specific critical thinking dispositions required for statistics versus mathematics, we raise the following questions:

- What challenges do mathematically successful students face in transferring their success in mathematics to statistics?
- How does the notion of subject-specific critical thinking dispositions explain the nature of these challenges?

In seeking answers to these questions, we discuss in the following section the issues cognitive psychologists have explored in the transfer of subject-specific critical thinking dispositions.

Literature Review

Subject-Specific Critical Thinking: There are some studies that have explored issues of general critical thinking disposition (Ennis, 1987; Sternberg, 1985). Also, as different problem-solving strategies are required in different subjects, some cognitive psychologists have explored critical thinking disposition with regard to subject matter knowledge (diSessa, 1987; Ennis, 1989; Even, 1993), which is referred to as *subject-specific critical thinking disposition*. Ennis (1989) set forth three principles to establish the relationship of subject matter knowledge to the development of subject-specific critical thinking. The first states that “knowledge about a topic is ordinarily a *necessary* condition for thinking critically in the topic” (p. 6). The implication is that it is imperative to acquire subject matter knowledge in order to be able to develop subject-specific critical thinking disposition. The second states that “simple transfer of critical thinking dispositions and abilities from one domain to another domain is unlikely” (p. 7). Under this principle, subject-specific critical thinking can be transferred to another subject only when there is sufficient practice in relevant domains and instruction that focuses on transfer. The last principle states that any general critical thinking instruction is not likely to be effective in developing subject-specific critical thinking. Unlike the first two, this principle is controversial and supported only by strong domain specificists (Ennis, 1989).

Subject-Specific Critical Thinking in Mathematics and Statistics: Aligning with Ennis’s (1989) views on subject-specific critical thinking disposition, a series of research studies has explored subject-specific critical thinking skills in mathematics and statistics. Aizikovitsh-Udi (2011) explored the effect of incorporating critical thinking training in a probability course. Her following study investigated if critical thinking skills depend on the content and the subject-specific concepts in that particular content (Aizikovitsh-Udi, 2012a). In this study, she claimed that “the construction and teaching of critical thinking skills are determined by specific contents and tasks the teacher uses” (p. 7). Recently, Aizikovitsh-Udi (2012b) extended the scope of her previous studies to explore how statistical literacy is linked with critical thinking skills.

Even though Aizikovitsh-Udi’s studies are valuable in that they incorporated critical thinking skills into the study of acquiring statistical knowledge, their focus was limited to comparing general critical thinking with statistical thinking. It still remains unknown what critical thinking skills are required in the study of statistics, how the development of the problem-solving skills in statistics advances the learning disposition specific to the discipline and the overall dynamics of statistics study, and how these processes in statistics contrast with those in mathematics. Keeping in mind Ennis's (1989) first principle, it is important to investigate how the different natures of the subject matter knowledge for mathematics and statistics require different problem-solving skills/strategies as the first step to answering the research questions. In the next section, we define critical thinking disposition and present our view on the transfer process between two different domains in relation with subject matter knowledge.

Theoretical Framework

In the first part of this section, we present the aspects of critical thinking skills that are relevant to the purpose of this study and then draw on these aspects to give our definition of critical thinking disposition. Then we explain our view on the process of how subject-specific critical thinking disposition transfers from one subject to another subject.

Conditions for Developing Critical Thinking Disposition: Halpern defines critical thinking disposition as deliberate use of skills/strategies that increase the probability of a desirable outcome (1998). Similarly, we define subject-specific critical thinking disposition as deliberate use of skills/strategies that increase the probability of a desirable outcome within a given subject. To develop this definition, one needs to satisfy certain conditions relevant to personal characteristics as a learner. Aligning with the views of Halpern (1998) and Facione, Sánchez, Facione & Gainen (1995) about the attitudes that a critical thinker exhibits, we consider that subject-specific critical thinking disposition is a habitual attitude towards the subject that can develop in the presence of the following three characteristics: (1) *eagerness* to immerse oneself in conceptually challenging tasks, (2) *flexibility* to apply problem-solving strategies developed within the study of a subject to problems that require the same strategies but in a new or different context for a different subject, and (3) *willingness* to discern the necessary critical thinking skills from the unnecessary ones. We believe that a learner's having developed a critical thinking disposition specific to mathematics transfers to his or her critical thinking disposition specific to statistics as he or she develops these characteristics.

Transfer of Subject-Specific Critical Thinking Disposition between Domains: Ennis (1989) categorized views on how to develop subject-specific critical thinking. For example, from the *general perspective*, critical thinking abilities and dispositions can be taught "separately from the presentation of the content of existing subject-matter offerings, with the purpose of teaching critical thinking" (p. 4). The *infusion perspective* holds that "deep, thoughtful, well understood subject matter instruction in which students are encouraged to think critically in the subject, and in which general principles of critical thinking dispositions and abilities are made explicit" (p. 5). We hold the general perspective, mixed with the infusion perspective in the sense that we advocate Ennis's first two principles.

As discussed earlier, mathematics and statistics are different in nature, and thus, demand different types of problem-solving skills and strategies. The implication of this, from our perspective, is that the subject-specific critical thinking dispositions developed in studying mathematics may not be automatically transferred to statistics. To explore this issue, it would be necessary to design instructions and practice that focus on transfer. Before we move to this study's methodology, we wish to note that we distinguish critical thinking disposition from the ability to think critically: "Some people may have excellent critical-thinking skills and may recognize when the skills are needed, but they also may choose not to engage in the effortful process of using them. This is the distinction between what people can do and what they actually do in real-world contexts" (Halpern, 1998, p. 452).

Method

The data for this case study with a single participant, Ian, are drawn from a larger study that explored student understanding of statistical concepts in two introductory statistics classes at a public research university. While the curricular organization of the courses conformed to those typically found in reform-oriented classrooms, the instruction itself was essentially traditional. The instructors had almost total responsibility for the classroom activities, and the content was delivered primarily via lecture.

We used a phenomenological approach to collect data, the process of which was conducted in two steps: a survey assessment and a follow-up interview. For the survey, we developed a fourteen-item assessment. Some of these items were modified from Assessment Resource Tools for Improving Statistical Thinking, developed by faculty members of the University of Minnesota in 2006. The rest of the items were developed by our research team. The entire survey is available by request from the first author. The assessment items sought to evaluate student understanding of what the symbols represented and their conceptual understanding primarily via their symbolic representations. The intent of the interview process was to identify how students' understanding of symbolic representations and their level of symbolic fluency potentially impacted their understanding of certain symbol-oriented concepts. The interviews were conducted immediately after the survey. Based on their work on the content survey, the eight students appear to range from low-achieving to high-achieving in statistics.

All interviews were audio-recorded and transcribed. For coding, each utterance was assessed to examine the information it gave about symbolic understandings. Within each transcript, we categorized and summarized the utterances that were deemed informative understandings by the type of concepts and connections they described with their symbolic understanding. We read within and across categories to develop conclusions. During the interviews, the grounded theory approach was blended in, to observe any interesting phenomenon with regard to the students' understanding of descriptive statistics. Both the survey and the interview were analyzed qualitatively. One thing we found was a stark contrast between Ian and other students as to how they understood the mathematical concepts that underlie statistical expressions and how they conceived of their statistics class. These findings motivated the authors to write this paper.

Result

The analysis of data informed us that Ian struggled in grasping some of the fundamental statistical concepts. We differentiate between those concepts that directly transfer from mathematics, such as symbolic manipulations and computations, and those that are statistical in nature and do not have exact analogs in mathematics. This was a surprising result because Ian had achieved A's in all his mathematics courses. The findings from the analysis of the data suggest what kinds of challenges mathematically strong students such as Ian may face by shedding light on how the critical thinking disposition favorably developed for learning mathematics could run counter to learning statistics. There were three specific main findings:

1. Strong Inquisitive Learning Disposition: During the interview, we found that Ian had a strong learning disposition for clarifying any confusion; he would not move on to solve the problems until he clarified the confusion. For example, Ian said, " I didn't really understand this. The highest governor's salary. Are they saying – … When they said the highest governor's salary, does that mean the highest ever reached?" Ian then asked questions to the interviewer to ensure he understood the symbols. Ian's inquisitive disposition was more explicitly revealed in the following claim about his statistics class, " … when you don't have that basic, basic stuff, it's, everything that comes after, you just struggle to try to put pieces together, all at the same time." This claim reveals Ian's view on how learning takes place as well as his inquisitive disposition.

2. Strong Understanding of Mathematical Concepts: The data also show evidence that Ian's understanding of underlying mathematical concepts of statistical expressions was exceptionally strong. For example, for the question, "Describe the distribution of x-mu in terms of the mean and the standard deviation as opposed to the distribution of x-μ where x follows a normal distribution," Ian claimed, "the center would still be zero. But the standard deviation would be sigma, because you forgot to divide. … our standard deviation would be sigma, instead of being one." For a subsequent question, "If you did x/σ-μ instead of (x-μ)/σ to obtain the z-score, would it matter?", Ian claimed, "Yes, it matters. Because you have to subtract μ divided by σ, because if you divided, if you do the shift first, by μ, you're centering it at zero." This claim shows that Ian understands the dynamics of the algebra that underlies the expression for the z-score. Among the eight students, Ian was the only student who provided the correct answers.

3. Failure of Transfer to Critical Thinking Disposition Specific to Statistics: During the interview, Ian showed evidence of successfully applying mathematical concepts in the context of probabilistic settings. To a question of how one can convince someone that something is wrong, Ian claimed, "If I had an example, I could show someone that it's definitely wrong." This claim shows that he knows a proof strategy often used in mathematics and is ready to use it in the given context. Ian further showed evidence of well-developed critical thinking strategies specific to mathematics. One question stated that in a university, 75% of the students are male and 25% are female; 5% of the males and 15% of the females own a car. The question asked whether we can conclude that 20% of the students in the university own a car. When the question was rephrased, without any instruction, as "Would you say that it's between 5 and 15, or would you say it's below 5, or would you say it's above 15?" Ian stated, "I would say it's between 5 and 15. Probably around 7%?" The claim implies that Ian grasps the mathematical concept of weighted average, which is primarily a concept that requires an a-contextual calculation.

In contrast, Ian showed weaknesses in transferring mathematical concepts to statistical contexts. For example, to the interview question, "What is a variable?" Ian replied, "I've always just seen variables as, like, things that could change, kind of, I'm thinking like algebra." This shows that he knows the definition of a variable in the mathematics context. But he was unable to transfer this mathematical concept to a statistical context until he was given instruction. The conversation went on as follows:

> Interviewer: Yeah, that's a good point. So, what are the things there, then, that could change?
> Ian: Based on what, though?
> Interviewer: That, you have a population. OK? But the population is a population that's fixed.
> Ian: OK. So, μ can't change, but x-bar and x could change … given different samples. And then, μ and σ can't change, because the population, overall, will always be the population.
> Interview: But why did you pick x-bar and mu, in the beginning, there?
> Ian: Because I didn't understand that at all. I didn't know what we were looking at as, what was changing and what wasn't changing.

It is important to note that when Ian was interrupted with an instruction, he was quickly able to transfer his understanding of a variable in a mathematical context to the statistical context.

Discussion

We speculate that the first finding, that Ian has a strong inquisitive learning disposition, explains both his success in mathematics and his struggle with statistics. This disposition led to his strong understanding of mathematical concepts and must have allowed him to be successful in his mathematics classes. However, this strong inquisitive learning disposition could have hindered his transfer of that success to his statistics course. Under the current statistics curricula that emphasize statistical literacy, reasoning and thinking (Ben-Zvi & Garfield, 2004), students are expected to accept certain statistical expressions without fully understanding the underlying mathematical concepts. In mathematics courses, students are often taught the underlying concepts (or at least given access to them) when a formula is given. The first and third findings together imply that an inquisitive disposition doesn't necessarily support learning in current statistics curricula.

We now shift gears to a brief discussion of the three characteristics needed to develop subject-specific critical thinking in statistics, and use them as a lens to analyze Ian's case. First, a strong inquisitive learning disposition could lay a favorable foundation for developing the *eagerness* to immerse oneself in conceptually challenging mathematical tasks, but could run counter to developing *eagerness* towards complicated statistical tasks because the focus of the statistical tasks is on understanding the context of the tasks, determining what statistical tests are appropriate, conducting the related computation, and interpreting the outcome, but not on understanding the mathematical concepts that underlie the statistical expressions. Second, students with a strong inquisitive learning disposition could have limited *flexibility* in applying problem-solving strategies developed within the study of a subject to problems that require the same strategies in the new or different context of a different subject. Third, it is possible to train a student's *willingness* to develop necessary subject-specific critical thinking disposition if he or she is provided with instruction, as we consider Ian's meta-cognitive comment, "I never grasped what variables were considered, in stats," as a first step to developing such willingness.

References

Aizikovitsh-Udi, E. (2011). Integrating theories in the promotion of critical thinking in mathematics classrooms. In M. Pytlak, T. Rowland, & E. Swoboda (Eds.), *Proceedings of CERME 7*, (pp. 1034–1043). University of Rzesvow, Poland.

Aizikovitsh-Udi, E. (2012a). *Developing critical thinking through probability models, intuitive judgments and decision-making under uncertainty*. (Doctoral dissertation). Saarbrucken, Germany: LAP Lambert Academic Publishing.

Aizikovitsh-Udi, E. (2012b). *Connections between statistical thinking and critical thinking – A cast study*. International Congress of Mathematics Education, Seoul, Korea.

Ben-Zvi, D. & Garfield, J. (2004). *The challenge of developing statistical literacy, reasoning and thinking*. The Netherlands: Kluwer Academic Publishers.

Cobb, G. W. & Moore, D. S. (1997). Mathematics, statistics, and teaching. *American Mathematical Monthly, 104*, 801–823.

diSessa, A. A. (1987). Phenomenology and the evolution of intuition. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 83–96). Hillsdale, NJ: Erlbaum.

Ennis, R. H. (1987). A taxonomy of critical thinking dispositions and abilities. In J. B. Baron & R. J. Sternberg (Eds.), *Teaching for thinking* (pp. 9–26). New York, NY: Freeman.

Ennis, R. H. (1989). Critical thinking and subject specificity: Clarification and needed research. *Educational Researcher, 18*(3), 4–10.

Even, R. (1993). Subject-matter knowledge and pedagogical content knowledge: Prospective secondary teachers and the function concept, *Journal for Research in Mathematics Education, 24*(2), 94–116.

Facione, P. A., Sánchez, C. A., Facione, N. C., & Gainen, J. (1995). The disposition toward critical thinking. *The Journal of General Education*, *44*(1), 1-25.

Garfield, J., & Ben-Zvi, D. (2008). *Developing students' statistical reasoning: Connecting research and teaching practice*. New York, NY: Springer.
Halpern, D. F. (1998). Teaching critical thinking for transfer across domains. *American Psychologist, 53*, 449–455.
Moore, M. & Notz, W. (2005). *Statistics: Concepts and controversies*. New York, NY: Freeman.
Rossman, A. J., Chance, B. L., & Medina, E. (2006). Some important comparisons between statistics and mathematics, and why teachers should care. In G. F. Burrill (Ed.), *Thinking and reasoning about data and chance: Sixty-eighth NCTM yearbook* (pp. 323–334). Reston, VA: National Council of Teachers of Mathematics.
Sternberg, R. J. (1985). Critical Thinking: Its Nature, Measurement, and Improvement. In F. R. Link (Ed.), *Essays on the intellect* (pp. 45-65). Alexandria, VA: Association for Supervision and Curriculum Development.
Swartz, R. (1992). Critical thinking, the curriculum, and the problem of transfer. In D. Perkins, J. Bishop & J. Lochhead (Eds.), *Thinking: Proceedings of the Second International Conference* (pp. 261–284). Hillsdale, NJ: Erlbaum.
Wild, C., & Pfannkuch, M. (1999). Statistical thinking in empirical inquiry. *International Statistical Review, 67*(3), 223–265.