Perspectives that some mathematicians bring to university course materials intended for prospective elementary teachers

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The undergraduate mathematical preparation of elementary teachers often occurs through mathematics departments. This study looks at the issue of teacher education as a site for collective work between mathematicians and mathematics educators. It works on a dual agenda of understanding what is involved, on the one hand, in using instructional support materials, and on the other hand, in creating usable instructional support materials. We analyzed three mathematicians' reviews of materials intended to teach *mathematical knowledge for teaching* to prospective elementary teachers. The analysis suggests that the mathematical issues most salient to these mathematicians concerned the coherence of the mathematical curriculum to be taught, the use and choice of mathematical representations, and what features of mathematical objects to make explicit. We discuss implications of this observation for development of materials for the mathematical preparation of teachers.

Key words: Mathematical Knowledge for Teaching, Teacher Education, Mathematicians, Mathematics Faculty

1. Introduction

The undergraduate mathematical preparation of elementary teachers often occurs through mathematics departments (Masingila, Olanoff, & Kwaka, 2012). There is wide agreement that content preparation of future teachers should connect to content demands of teaching (Conference Board of the Mathematical Sciences, 2012). Yet the educational work in which these mathematical demands arise – such as linking different representations of multiplication of fractions to each other, or leading a discussion on using the definition of fraction to place fractions on the number line – may involve activities on topics or with manipulatives not typically part of a mathematician's professional training. Preparing teachers to meet the mathematical needs of their work thus calls for collaboration between mathematics educators and mathematics faculty.

Mathematical knowledge for teaching (MKT) is defined as the knowledge required to meet the mathematical demands of teaching (Ball, Thames, & Phelps, 2008). The study reported in this proposal looks at the issue of teaching MKT, especially as a site for collective work between mathematicians and mathematics educators. It works on a dual agenda of understanding what is involved, on the one hand, in using instructional support materials for teaching MKT, and on the other hand, in creating usable instructional support materials for teaching MKT. The research questions we focus on are:

- What perspectives do mathematicians bring to interpreting and enacting instructional support materials intended for teaching MKT?
- What is salient to mathematicians regarding the work of teaching MKT and instructional goals for teaching MKT?

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We examined these questions in the context of the mathematical preparation of elementary teachers, in which mathematics courses are frequently based on materials by mathematics educators and taught by mathematics faculty. Our results shed insight on identifying key considerations for using and creating usable instructional support materials for teaching MKT.

To investigate the research questions, we analyzed reviews by 3 mathematicians of instructional support materials, for 10 hours of instruction to prospective elementary teachers in mathematics courses taught through mathematics departments, produced by mathematics educators. These materials were intended by their developers to teach MKT. We asked reviewers to comment on the quality of the materials in terms of their fit with the developers' stated instructional goals. We asked the reviewers to assess the fit of the materials with the goals as a way to position them as potential instructors interpreting these materials and their goals for what was entailed in enacting the instruction. This study focuses on insight yielded from the mathematicians' reviews. Though we also solicited reviews by 3 mathematics educators, these were primarily used as potential for contrast rather than as objects of examination.

2. Conceptual basis

Though mathematicians are experts in content, the way they typically interact with elementary content such as fractions likely differs from how they would need to interact with fractions if they were instructors for mathematical knowledge for teaching fractions. Interaction with content is only one part of the complex work of teaching; teaching includes interactions with students and their interactions with content. These interactions are captured by the conception of teaching as the management of interactions amongst teacher, students, content, and the environment (Cohen, Raudenbush, & Ball, 2002), constituting an "instructional triangle". In the context of teacher education that this study concerns, we use "instructor" to designate university instructors in place of "teacher", and "teachers" in place of "students" (Figure 1).

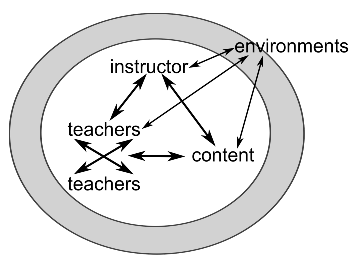

Figure 1. An instructional triangle for teacher education.

We take *mathematical knowledge for teaching* (MKT) to mean the "mathematics needed to perform the recurrent tasks of teaching mathematics to students" (Ball, Thames, & Phelps, 2008, p. 399). Besides understanding student thinking based upon student productions, MKT also includes the mathematics for evaluating or creating precise yet accessible definitions for concepts such as fractions or polygons, selecting and sequencing which students' ideas, when shared in discussion, would support meeting a particular mathematical goal, or designing assessment problems to elicit particular student conceptions or misconceptions. As well, MKT encompasses pedagogical content knowledge such as being able to anticipate what students are likely to think and what they will find confusing, being facile with different representations of mathematical concepts, and evaluating the affordances and limitations of examples to illustrate concepts. MKT has been linked to quality of instruction and positive student outcomes (e.g., Hill, Rowan, & Ball, 2005; Kane & Staiger, 2012).

3. Design of analysis and data

We analyzed 3 mathematicians' reviews of instructional support materials intended for teaching MKT to prospective elementary teachers. The instructional support materials included detailed lesson plans for 10 sessions of instruction, instructional goals for each session, and mathematical and pedagogical commentary directed to the instructor. The materials also included records of practice such as video episodes of teaching and sample elementary student work.

The 3 mathematician reviewers each had experience with mathematics courses for prospective elementary teachers. We use the pseudonyms Abbott, Barrett, and Carter for the mathematicians. Additionally, we solicited reviews by 3 mathematics educators to provide contrasting perspectives.

The instructional goals for the materials, written by the materials developers, were based on the aim to develop instructors' understanding of the notion of MKT alongside prospective teachers' understanding of MKT for elementary grade teaching. The materials featured sessions on the concept of fraction, operations on fractions, and placement of the fraction on the number line. We asked reviewers to assess the quality and fit of the materials with the instructional goals regarding the:

- structure, clarity, and treatment of the content
- connection to the mathematical demands of teaching
- opportunities for teachers' learning of mathematical knowledge for teaching.

The focus on content and implementation with respect to goals was intended to position reviewers as potential instructors interpreting the materials and goals. We hypothesized that the focus on implementation would prompt reviewers to make sense of MKT as conveyed by the materials to the instructor and of the teaching of MKT to prospective teachers from the perspective of an instructor. The reviewers commented on the content as well as provided insight into other aspects of the management of instruction.

We parsed reviews into assertions made about the instructional support materials. In total, we parsed 130 assertions from the mathematicians' reviews and 73 assertions from the educators' reviews. (The educators' reviews were shorter than the mathematicians' reviews.)

To analyze assertions for their perspective on teaching, we used interactions of the instructional triangle as an initial coding scheme. We coded an assertion with an interaction if the assertion addressed that interaction. There were very few assertions specifically regarding the interaction between instructors and teachers or the mediation between teachers and content, and there were many assertions simultaneously discussing instructors, teachers, and content – frequently addressing dilemmas of teaching. Such assertions were coded together under the code, "work and dilemmas of teaching MKT". Additionally, there were many assertions about the content that teachers learned or ought to learn that did not discuss an instructional interaction. These were coded as corresponding to the content component of the instructional triangle. Ultimately, we used four codes, as shown in Table 1.

Table 1. Codes for reviewers' assertions.

Assertion code	Component(s) of instructional triangle
Work and dilemmas of teaching MKT	Instructor, teacher, and content
Views of mathematics and MKT	Instructor and content
Environmental/broader concerns	Environments
Mathematics that teachers should be learning	Content

Codes are shown in Figure 2 with their associated instructional component. Assertions could be associated with more than one code. For example, the mathematician Abbott described how he would discuss the concept of 3/4 with prospective teachers, and then wrote: "Though this is perhaps not the best way to introduce 3/4 to third-graders, it is, I believe, an important concept for all elementary teachers to grasp and come to grips with." This statement, along with the description of how he leads a discussion of 3/4, was coded as concerning the work of teaching (because it concerned his interaction with the teachers and the content of the concept of 3/4) as well as concerning content as Abbott asserts that this is a concept that teachers ought to learn.

This paper reports on assertions about the work and dilemmas of teaching MKT and views of mathematics and MKT. We analyzed assertions with these two codes for themes regarding the interpretation and enactment of materials, and analyzed the frequency of these themes in the mathematicians' and mathematics educators' reviews.

We emphasize again that this document focuses on reviews by mathematicians. The reviews by mathematics educators were primarily used as potential for contrast rather than as objects of examination.

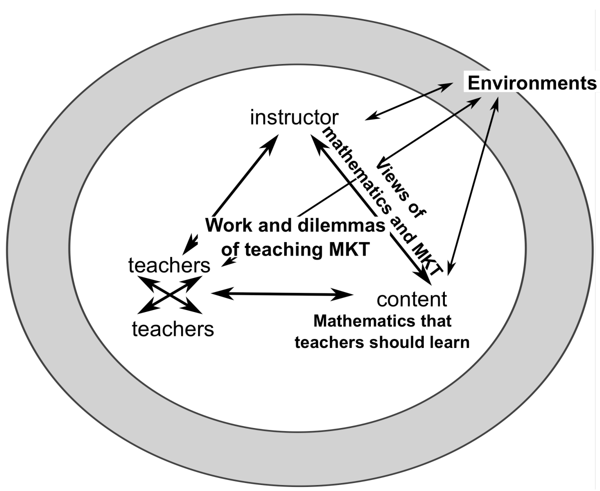

Figure 2. Codes and their corresponding components of the instructional triangle. Each review was parsed into assertions, and each assertion was coded with the component of the instructional triangle it concerned.

4. Results

Two themes regarding interpretation and enactment of materials that arose from our analysis of the reviews were: adapting or extending curricular content for prospective teachers' mathematical education, and pedagogical and mathematical issues influencing the way instruction is managed. We summarize and then illustrate the findings with an example from the reviews.

- **Adapting or extending curricular content.** Our analysis suggests that the mathematician reviewers would appraise the content of materials based on: prospective teachers' own mathematical knowledge and practice, the coherence and clarity of the mathematics, whether mathematical ideas were made sufficiently explicit, and how mathematical structure was used by the examples and exercises provided by the materials. These reasons for judging the merit of the content discussed by the materials, as representing the mathematics to be learned by future teachers, were most frequently cited in the assertions.

We note that the mathematical issues most salient to these mathematicians through these materials concerned the mathematical structure and coherence, as opposed to the ease or difficulty for teachers to learn a particular topic or the developmental needs of the teachers' future students, concerns more prominent to the mathematics educators who provided reviews. Both mathematician and educator reviewers were concerned with the prospective teachers' mathematical knowledge and practice.

- **Pedagogical and mathematical issues.** The issues most salient to the mathematicians – those they chose to comment on – were how representations were used to explain a mathematical concept, what ideas or features to discuss explicitly, allocation of instructional time, and how to sequence related but different concepts across a unit of instruction.

In contrast, some of the pedagogical issues most salient to the educators included choice of representations and how to connect different interpretations of one concept. Though mathematicians did also comment on choice of representations, their reviews contained more comments about the particular use of given representations than critiques of the collection of representations as a whole.

Illustration of some themes. We consider mathematicians' comments on the number line. These comments provide an instance of mathematicians' concern with the overarching mathematical structure of the materials, what ideas to make explicit, and how mathematical structure of an object is used. All three mathematicians commented on uses of number line to address perceived mathematical gaps or missed mathematical opportunities in the materials. For example, Abbott was concerned that the materials would leave prospective teachers without the understanding the 3/4 can be defined as the solution to the equation $4x = 3$. He chose to address this by developing an explanation using the number line, going into the meaning of multiplying and dividing by 4 as dilation symmetries of the number line. Barrett lamented the lack of emphasis on the number line, noting that the number line is a "key representational tool". Barrett stated several times that the relative emphasis on the part-whole definition of fraction as opposed to a number line conception is misplaced, and that he wished that discussion of the number line had come earlier. Although he does not elaborate further, he cites the IES Practice Guide (Siegler et al., 2010), which promotes the number line representation for its ability to illustrate connections between fractions, whole numbers, and percents; and describes student misconceptions arising from over-reliance on a part-whole conception. Carter asserted that the treatment of properties of the number line missed opportunities to articulate the mathematics fully, especially the informal description of the notion of infinitesimally close contained in the materials. As he noted, "It does not come close to the level of clarity, precision and accuracy that I think elementary teachers are capable of mastering and using in their classrooms."

5. Discussion

Abbott, Barrett, and Carter raised issues related to the number line and its mathematical properties and made fewer comments addressing how teachers might connect different conceptions of fraction to each other. Given the importance of elementary teachers' ability to help their students map representations to each other (e.g., Resnick and Omanson (1987)) mathematicians' sensitivity to the mathematical structure of particular representations could be leveraged in collective work between mathematicians and mathematics educators to yield curricular materials that would enable teachers to make more refined links between different representations. As well, though there are many comments about local mathematical issues (such as Carter's comments on the treatment of number line properties) and global mathematical issues (Barrett's comments about fraction conceptions), there are relatively few comments addressing connections between consecutive sessions, mathematically or pedagogically. Understanding the rationale behind the instructional design from session to session could as well be a site for collaboration between mathematicians and mathematics educators.

We cannot say that the perspectives in the reviews we examined are indicative of mathematics faculty in general, however the findings suggest that mathematics faculty who reviewed these materials were concerned with the mathematical structure of curricula to make sense of how to teach. We propose that it would be useful in advancing collaborations between mathematicians and mathematics educators to include discussions about how they approach making sense of the content preparation of teachers. It stands to reason that given the differences in their professional experiences working with teachers, different issues may be salient to them. Our experience listening to these reviews suggest that considerations for using and creating usable instructional support materials, for teaching MKT in mathematics courses taught in mathematics departments, could include elaborations of task enactments that connect in-the-moment decisions with how they support discussions about the mathematical structure of concepts, and that link the enactments with the overall structure of the materials.

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