# Using Disciplinary Practices to Organize Instruction of Mathematics Courses for Prospective Teachers

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One challenge of teaching content courses for prospective teachers is organizing instruction in ways that represent the discipline with integrity while serving the needs of future teachers—for example, choosing math problems that provide a logical development of a topic while also addressing mathematical knowledge for teaching. This paper examines the work entailed in structuring in-class work in mathematics courses for teachers. It argues that practices of teaching that are mathematical—such as representing ideas, grounding reasoning in mathematical observations available to the class, using definitions, or using mathematical language—can be used to negotiate mathematical and pedagogical aims, and therefore can be used to organize instruction of mathematical knowledge for teaching while simultaneously developing a disciplinary understanding.
*Key words:* Mathematics Teacher Education, Mathematical Quality of Instruction, Mathematics Task Framework

## 1 Introduction: Challenges of structuring courses for prospective teachers

Teachers must know subject matter for their work, and programs for prospective teachers often include courses on subject matter to address this need. This paper focuses on mathematics courses, often offered by mathematics departments, intended to be simultaneously relevant to mathematics as a discipline and to what is needed in the classroom.

Although there is limited examination of the relationship between mathematical knowledge for teaching, instructional quality, and student learning, existing studies reveal that measures of mathematical knowledge for teaching and instructional quality are significantly associated with positive student outcomes (Hill, Rowan, & Ball, 2004; Baumert et al., 2010; Rockoff, Jacob, Kane, & Staiger, 2011; Kane & Staiger, 2012). Thus mathematical knowledge for teaching is a natural candidate for what to teach to prospective teachers.

Yet, mathematical knowledge for teaching can be difficult to teach. When discussing mathematical knowledge for teaching, it can be easy to slide into mathematics not tied to teaching, or into instructional issues that are not mathematical. Because the elaboration of mathematical knowledge for teaching is the subject of current research, it is not codified in any standard form, making adoption as curricula difficult. Finally, the nature of mathematical knowledge for teaching, as mathematics not strictly for advancing the discipline, implies that pedagogical considerations for its teaching may differ from those for courses typically taught by mathematics instructors. Instruction of mathematical knowledge for teaching faces challenges in enactment and specification of content.

One aspect of these challenges is the course structure, from global curricular organization to in-class tasks and the accompanying demands for hearing and responding to ideas raised. Mathematical practices may offer a resource to meet the challenges. The structuring of courses with mathematical practices of teaching contrasts with, for example, using mathematical topics or techniques to organize coursework, as tables of contents of many textbooks would suggest. Most textbooks are organized by major theorems, computational techniques, or proof techniques. However, because mathematical knowledge for teaching intertwines mathematical and pedagogical considerations, a course structure based on disciplinary topics and techniques risks a slide into mathematics that is not as supportive of the work of future teachers. However, little is known about using mathematical practices of teaching to structure mathematics content courses for teaching.

In this paper, I examine the questions:

- What are affordances and challenges of using mathematical practices to organize instruction of mathematical knowledge for teaching?
- What is involved in effectively using mathematical practices to organize instruction in ways that engage mathematical knowledge for teaching?

Using the case of an instructor who used practices to organize his instruction, I argue that (a) mathematical practices provide a valuable resource for developing assignments that address mathematical knowledge for teaching; (b) challenges to engaging mathematical knowledge for teaching occur during task set up and implementation, especially in the absence of fine-grained knowledge of mathematical practices; and that (c) engaging mathematical knowledge for teaching requires the parsing of knowledge at two grain sizes: first, into a grain size that allows naming of entailed practices; and second, into a refined elaboration within practices. (Henceforth, I use *instructor* to refer to the "teacher", and *teachers* to refer to the prospective teachers who are the students of the instructor.)

## 2 Theoretical frameworks used

Shulman (1986) introduced the notion of pedagogical content knowledge as an amalgam of subject matter (such as mathematics) and pedagogy. Extending Shulman's work, Ball and her colleagues (e.g., Ball, Thames, & Phelps, 2008) conceptualized MKT as the knowledge of mathematics "needed to perform the recurrent tasks of teaching mathematics to students" (Ball et al., 2008, p. 399).

The recurrent work of teaching includes *using representations*; as Shulman (1986) observed, teaching requires knowing "the most useful ways of representing and formulating the subject that make it comprehensible to others" (p. 8). Representing mathematical ideas requires mathematical and pedagogical sensitivity. For instance, the most appropriate representation of a fraction—whether visual, verbal, symbolic, or another mode—depends on what students know, the purpose of the lesson, and what the representation suggests about the relationship between wholes and parts of fractions. The recurrent work of teaching also includes *explanations of mathematical reasoning*, in particular, *grounding the reasoning in evidence available to the class*. As Ball and Bass (2003) argue, the mathematical resources available to a class determine mathematical validity. For a statement to be valid in a community, its proof must be logically permissible according to the discipline and it must be grounded in observations accessible to the community.

Because representation and explanation occur across the teaching of many mathematical topics, I refer to them as examples of *mathematical practices of teaching*. I choose this phrase in analogue to "mathematical practices" entailed in doing mathematics. Mathematics is developed through practices (Kitcher, 1984); the recent Common Core Standards for Mathematical Practice (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010) are one representation of disciplinary entailments. I also use the phrase "practices" in the sense of "practices of a profession", for example, as discussed by Grossman et al. (2009).

## 3 Methods of analysis

Teaching is the management of instruction, which consists of the interactions among the teacher, content, students, and environment (Cohen, Raudenbush, & Ball, 2002). An analysis of teaching is informed by analysis of interactions. This study focuses on interactions among teachers, the instructor, and the content of mathematical knowledge for teaching (MKT). I analyzed (a) records of practice for a mathematics course for prospective elementary teachers, and (b) interviews with the instructor for that course. I focus the analysis on two episodes featuring one task each, examining both the nature and role of the tasks used and the qualities of the interactions in the classroom.

In teaching, tasks serve as a medium for communicating the nature of mathematics to learners. Using the language of "instructor" and "teachers", a task focuses teachers' attention on a particular idea; it is characterized to include what teachers are expected to produce, how they are to produce it, and the resources available (Stein, Grover, & Henningsen, 1996). Stein et al. (1996) introduce the *mathematical task framework* (MTF) to understand the relationship between task set up, implementation, and resulting learning.

The deployment of practices influences the quality of instruction. For instance, making explicit links between representations and ideas may help students connect familiar and more technical representations, and asking for explanations of why and not just how procedures work may help students generalize thinking. These kinds of instances support the "richness of the mathematics" as measured by the Mathematical Quality of Instruction (MQI) instrument, which is associated significantly to positive student outcomes and to MKT (Hill et al., 2008; Kane & Staiger, 2012).

To understand the challenges of teaching mathematics useful for prospective teachers, I conducted separate analyses using MTF and MQI. I used MTF to analyze relationships between task characteristics and teachers' learning and to design interview questions addressing the instructors' goals, knowledge, and dispositions. I analyzed interviews for themes regarding his perceptions of the affordances and challenges of using mathematical practices to organize his instruction.

The second analysis of the data was based on MQI. I adapted the codes from MQI to analyze representations and explanations by the instructor and teachers in the course. I noted high-quality instances of representations and explanations, as well as instances where high-quality instruction might have occurred but did not. I wrote descriptions of these instances and analyzed them for themes regarding affordances and challenges of using representations and grounding reasoning. Finally, I compared the analyses for how the extracted themes contrasted and complemented each other in responding to the research questions.

## 4 Data

I used records of practice including video, course curricula, instructor notes, and interviews. The two analyzed episodes occurred in the first and third month of the course. In one episode, the instructor is facilitating discussion about representing 7/3 and the whole on the number line (*Task 7/3*). In the second episode, the instructor and teachers are explaining representations of $1/3 \div 1/5$ and connecting them to meanings of division (*Task $1/3 \div 1/5$*).

## 5 Results

The analyses suggests that mathematical practices provide a resource for developing in-class, homework, and exam tasks that otherwise might not have been created. The data came from the first year the instructor used mathematical practices to organize instruction. To the instructor, these questions from this year represented MKT more genuinely than those in any previous year. This was also the first year in which he felt that he truly addressed mathematics serving the teachers' future work. He highlighted several assignments that illustrated the impact of using mathematical practices, including one asking teachers to "explain how to see the dividend, divisor, and quotient" in a variety of representations, followed by asking what interpretation of division was represented. This assignment has rich potential for teacher learning and high-quality instruction: it affords opportunities for high-demand processes such as justifying and interpreting, and high-quality interactions such as linking representations with mathematical ideas.

A key challenge is managing instruction to realize potential for learning. The discussion on the homework assignment described above, part of the setup for Task 1/3÷1/5, exhibited substantive mathematical engagement by teachers and high-quality representation and explanation by both teachers and the instructor. In contrast, the instruction on Task 7/3 featured lackluster engagement and lesser quality of instruction—despite its rich potential. As Stein et al. (1996) observed, the potential for a task is only potential. The instructor had envisioned high cognitive demand for both tasks, but this was realized in only one. The analyses suggest that part of what it takes to organize instruction effectively is parsed knowledge of practices. During Task 7/3, the instructor asked teachers how they "liked" the representation; and he was unable to initiate substantive discussion about features. But suppose that explicit, finely parsed constituents of the practice of representation had been available—for instance, it was known that representation entails linking across different representations, linking to mathematical structures, or that to work with a representation one must first be able to discuss its salient features. Then perhaps the enactment of this task might have been sharper: instead of asking what appeals about a representation, the instructor might have asked, "What aspects of the representation make the whole less clear?" The proposed paper discusses potential elaborations of the mathematical practices of representing ideas and grounding reasoning.

## 6 Implications

The argument advanced here—that mathematical practices of teaching can be used to organize instruction of courses for teachers—may apply to other disciplines such as science and English language arts, and to other types of courses. In fact, decomposing practice has been used to structure methods courses (Kazemi, Lampert, & Ghousseini, 2007). I argue that decomposing the mathematical practice of teaching into its constituent practices could be used profitably for content courses; the practices should be disciplinary practices of teaching that support pedagogy rather than pedagogical practices of teaching that support the discipline. To examine to what extent this idea may cross subject matter boundaries, one would need to have an idea of the recurrent work of teaching a particular subject matter. This would involve identifying practices of an appropriate grain size for organizing instruction (for example, perhaps "explicit use of the scientific method" represents too large a grain size, whereas "organizing data to support hypothesis testing" represents a more appropriate size), selecting and sequencing domain topics and techniques that best support the learning of these domain practices of teaching, and then making empirical studies of how these practices inform task enactment. This approach may offer the advantage of promoting coherence between methods and content courses: if both types of courses were structured in a practice-based way, then practice could provide a commonplace for the improvement and understanding of the complex work of teaching, and how learning of teaching practice can build upon its constituent practices.

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