SCALING UP REINVENTION: DEVELOPING A FRAMEWORK FOR INSTRUCTOR ROLES IN THE CLASSROOM

Junguen Park
University of Delaware
jungeun@udel.edu

Jason Martin
University of Central Arkansas
jasonm@uca.edu

Michal Oehrtman
University of Northern Colorado
michael.oehrtman@unco.edu

Studies have shown that students have difficulty with the concept of limit, especially when reasoning about formal limit definitions. We conducted a five-day teaching experiment (TE) in a second semester calculus classroom in which students were asked to reinvent a formal sequence convergence definition. Oehrtman et al. (2011) detailed how pairs of students reinvented sequence convergence definitions but did not attempt the same instructional heuristic in the classroom. Our analysis focused on the instructor prompts and the TE students' subsequent group discussion through their use of key words and visuals in revising their definition. An interview with the instructor was conducted to investigate his intention of using specific prompts and his thinking about the TE group's choice of words and visuals. In our preliminary analysis, we found that the roles of the instructor were extended beyond those roles previously reported as roles for facilitators with pairs of students.

Keywords: Guided reinvention, Sequence, Limit, Calculus, Classroom Discourse, Instructor Role

Much of the previous literature on limits has focused on students' misconceptions within their informal understanding of limits (e.g., Bezuidenhout, 2001; Cornu, 1991; Davis & Vinner, 1986; Monaghan, 1991; Tall, 1992; Williams, 1991). Students given a formal limit definition can have great difficulty making sense of the intricacies of that definition (Artigue, 2000; Bezuidenhout, 2001; Cornu, 1991; Tall, 1992; Williams, 1991). Recent studies have used empirical results to detail how students come to understand formal limit definitions (Cottrill et al., 1996; Martin et al., 2011; Oehrtman et al., 2011; Roh, 2008; Swinyard, 2011; Swinyard & Larsen, 2012). This paper attempts to add to this body of research by exploring the nature of instructors' role as facilitators of students' discussion in a classroom where students were asked to reinvent a formal definition of sequence convergence by focusing on instructor prompts. We ask, what are the roles of the instructor as a facilitator in a classroom where the instructor is implementing guided reinvention for a formal definition of a limit concept?

Literature Review

Within the theory of Realistic Mathematics Education (RME; Freudenthal, 1973), guided reinvention is an instructional heuristic that aims to position students in experientially real contexts to support the emergence of formal mathematics through students progressively constructing the mathematics for themselves (Gravemeijer, 1998, 1999). Gravemeijer (1999) stated that the "idea is to allow learners to come to regard the knowledge they acquire as their own private knowledge, knowledge for which they themselves are responsible" (p. 158). Concerning the reinvention of formal limit definitions, recent studies have detailed challenges students face and the importance of engaging those challenges (Martin, Oehrtman, et al., 2012; Oehrtman et al., 2011; Swinyard, 2011; Swinyard & Larsen, 2012), the role of quantitative reasoning (Martin, Cory, et al., 2012), and how computer generated dynamic graphs might

support decisions made during reinvention (Cory et al., 2012). In particular, Oehrtman et al. (2011) has described the reinvention process as an iterative refinement process (IRP). Within this process, students write a definition, evaluate their definition against a rich collection of examples and non-examples, acknowledge problems with their definition, discuss potential solutions, and revise their definition, thereby initiating another iteration.

The previous studies with pairs of students described three main roles of a facilitator as nudging students forward to the next logical phase of the IRP (Steering the Ship), producing conflict (Conflict Producer), and providing timely solutions (Solution Provider) (Oehrtman et al., 2011). When students persist in overlooking a problematic issue with their evolving definition, facilitators can act as *conflict producers* by asking students to interpret their definition applied to a particular graphical example that their definition does not appropriately capture. After students wrestle with a problem for a significant time and have sufficient understanding of solution elements but remain unable to come to a satisfactory resolution, facilitators might act as *solution providers* while preserving the students' intellectual ownership of the process.

These previous guided reinvention studies were conducted with pairs of students while acknowledging that whole class implementation of such activities needed further research. Therefore, we implemented a guided reinvention approach to support students in constructing a formal limit definition as a part of a series of lessons implemented within a second semester calculus classroom. Video data revealed differences between the interview and classroom settings in terms of the role of the facilitator (instructor in the classroom), and the nature of interaction between students and facilitator. To address these differences in a systemic way, we are starting from these three facilitator roles described above, and have identified subcategories focusing on instructor prompts for the whole class and small group (Table 1).

Theoretical Framework & Method

We conducted a five-day TE in a second semester calculus classroom with 11 students at a medium-sized public university. Over the past three years, the instructor, who is the second author, has been both a researcher and facilitator with three guided reinvention teaching experiments with pairs of students. A *developmental research* approach "to design instructional activities that (a) link up with the informal situated knowledge of the students, and (b) enable them to develop more sophisticated, abstract, and formal knowledge, while (c) complying with the basic principle of intellectual autonomy" was adopted (Gravemeijer, 1998, p. 279). The students were first asked to create examples and non-examples of graphs of sequences converging to 5. Students were then asked create a formal definition for sequence convergence by completing the statement, "A sequence converges to 5 provided..." The instructor then guided the class through the IRP. There were two groups of four students and one group of three students. The activity of one group of four students was video and audio recorded (referred to as the TE group) and a second video camera captured whole class interactions with the instructor.

In our preliminary analysis of instructor's roles while conducting a guided reinvention in the classroom, we are developing an analytical framework for instructor prompts through an open coding of classroom videos (Strauss & Corbin, 1990). We especially focused on instructors' use of words and visuals, and his choice of students' words and visuals (Sfard, 2008).

Emerging Results

During our preliminary analysis, we found new categories of instructor roles in addition to providing more detail to previously identified facilitator roles. The current framework is shown in Table 1. Examples for subcategories were direct quotes from classroom videos.

Table 1

Categories of Instructor's Role

Category	Subcategory	Description	Example
Framing the Task	Inclusion (IN)	Explaining what needs to be discussed in IRP	"Your definition is supposed to be precise and concise, and capture all the things it should, all the examples, and exclude all the non-examples" (Day 2).
	Exclusion (EX)	Explaining what is not going to be discussed	"We don't question ... the convergence of these graphs" (Day 1).
Steering the Ship	Procedure (PC)	Telling students what to do next	"I want you to take another stab at your definition" (Day 1).
	Intra-Group Activity (IntraG)	Leveraging multiple- person activity within the group: Asking students to move from individual activity to group activity	"Exchange what each of you did… discuss and then make another attempt at a group definition" (Merging 4 individual definition into 1) (Day 2).
	Inter-Group Activity (InterG)	Leveraging multi-group activity in the whole class: Giving students an opportunity to see what other group was doing	"You're going to see everybody else's [definition], some problems they wrestled with and how they may have resolved some. If you see something that you think, oh, that might be a good thing to adopt, or oh, we don't want to" (Day 3).
	Focus on Visuals/Words (FI)	Directing student focus toward specific visuals or words provided by instructor (e.g., a graph, particular attributes of a graph)	"How would you describe the end behavior of [Graph]?… How many dots above and below each time [on Graph G]"? (Day 1)
	Focus on Definition (FD)	Directing student focus toward their use of specific visuals or words in their definition	"Where did the 'ultimately' [included in a TE's definition] occur like on Graph B?" (Day 2)
	Presenting (PR)	Directing students how to talk/present	"I want you to present your problem, not necessarily your definition" (Day 3).
Conflict Producer	Selecting Examples (SE)	Ask students to apply the definition to selected graphs	"How does your definition work for [Graph] A?" (Day 3)
	Third Person (TP)	Ask students to responds to the third person's question	"The student can look at this, and say 'I'm not sure exactly what this 'approach' is. I'm not sure if it captures this B graph?'" (Day 1)

Table 1 (continued)

Categories of Instructor's Role

Category	Subcategory	Description	Example
Conflict Producer (continued)	Other Group (OG)	Let student in one group talk to another group	A student in one group to another group "It [Graph B] does not always approach 5." (Day 1)
	Highlighting Process (HP)	Noting how students applied their definition in ways not captured by their definition	With hands spread over graphs, "What did you always draw first?... Do you feel like... your definition captures that?" (Day 4)
Solution Provider	Direct Solution (DS)	Correcting/clarifying meaning	"You can have error bounds without necessarily having over and under [estimates]. Error bounds just means, bounding the error" (Day 2).
	Student Source (SS)	Selecting one students' work among TE students	"I heard, [student's name], you saying something, but I don't see it incorporated here." (Day 2)
	Group Source (GS)	Selecting a group's work among other groups	While addressing whole class, "So you decided error bound. [Student's name], could I get you to quickly illustrate? I mean a lot you've done here [on group's board]." (Day 3)

The first main category includes instances where the instructor explicitly explained what will or will not be part of the IRP. The second category, steering the ship, includes cases when the instructor moved students forward in the IRP by explaining the procedure or asking students to focus on specific words and visuals in his examples, or in students' own definitions. The third category included the instances where the instructor attempted to produce conflict for students by selecting examples to which a current student definition did not apply, choosing words in their current definition that may cause conflict, using a third person who disagrees with their definition, asking a student in other group to speak who had a different view about a problem that a group was trying to resolve, and highlighting a process being used to apply their definition that was not captured in their definition. Codes SE and HP in Conflict Producer involve various instructor prompts for students to see the consistency/inconsistency between their written definition and their application in the examples/non-examples. Usually, the cases for SE identified when the instructor asked students to apply their written definition on the graphs, and the cases for HP identified when instructor pointed out the inconsistency between the students' written definition and their graphical explanation after the students applied their definition to the graphs. In the last category, solution provider, the instructor either provided an explicit definition of words for TE students to use or promoted one student's or another group's work to help resolve a problem. We note that the instructor could use non-TE groups' work either to produce a conflict (OG) and provide solution (GS). Because the difference between these two codes based on the video data only were subtle, such instances were complemented by the interview with the instructor about his intention on using other groups' work.

As this study built on previous research with pairs of students (e.g., Oehrtman et al., 2011) to groups of four students within a whole class setting, new detail has emerged in describing instructor roles. The subcategories indicate that having four students in the TE group and other

groups in the classroom enabled the instructor to use other student work to produce conflict or provide solution. For example, we note that as a conflict producer, the instructor used the work of *another group* for TE students to realize problems in their definition. As a solution provider, instead of providing a crucial element of a formal definition, the instructor let *other groups* presents their definitions and problems to the whole class and provided TE students opportunities to adopt or reject other students' idea.

Discussion

Our preliminary analysis revealed various aspects of instructor and student discourse, which were different from interview settings. In particular, instructor's promoting an individual student's work or the work of another group made the nature of instructor-student and student-student interaction different from those previously reported with pairs of students. We anticipate that applying Table 1 as an analytical framework to our data will help us to better detail the nature of classroom discourse. In the future, we will attempt to apply this framework to address the following questions:

1) What are characteristics of the instructor's discourse during instructor-student interaction while complying with the principles of guided reinvention?
2) What are relations between the instructor's prompts and students' process of reinventing a formal definition?

To address the first question, we will investigate various aspects of instructor discourse using our categories of instructor's role, including the frequency of each subcategory and changes in the frequencies over time. We will also explore the nature of instructor-student and student-student discussion after a certain instructor prompt to identify types of instructor prompts that support productive student activity in the sense of progressing through the IRP. In our presentation, we will seek feedback on our categorization of instructor's roles and the systematic characterization of instructor-student classroom interaction. Because our primary goal was implementing a guided reinvention in a classroom setting, we will also solicit suggestions on how we can provide better resources for instructors who plan to use this approach in their classroom.

Acknowledgement

References

Artigue, M. (2000). Teaching and learning calculus: What can be learned from education research and curricular changes in France? In E. Dubinsky, A. Schoenfeld, & J. J. Kaput (Eds.), Research in collegiate mathematics education IV (pp. 1–15). Providence, RI: American mathematical society.

Bezuidenhout, J. (2001). Limits and continuity: Some conceptions of first-year students. *International Journal of Mathematical Education in Science & Technology, 32*(4), 487-500.

Cornu, B. (1991). Limits. In D. O. Tall (Ed.), *Advanced Mathematical Thinking* (pp. 153-166). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Cory, B., Martin, J., Oehrtman, M., & Swinyard, C. (2012). The use of dynamic visualizations following reinvention. In S. Brown, S. Larsen, K. Marrongelle, & M. Oehrtman (Eds.), *Proceedings of the The Fifteenth Conference on Research in Undergraduate Mathematics Education*, Portland, OR: Portland State University.

Cottrill, J., Dubinsky, E., Nichols, D., Schwingendorf, K., Thomas, K., & Vidakovic, D. (1996). Understanding the limit concept: Beginning with a coordinate process schema. *Journal of Mathematical Behavior, 15*, 167-192.

Davis, R. B., Vinner, S. (1986) The notation of limit: Some seemingly unavoidable misconception stages. The Journal of Mathematical Behavior, 5, 281-303.

Freudenthal, H. (1973). *Mathematics as an Educational Task*. Dordrecht, The Netherlands: Reidel.

Gravemeijer, K. (1998). Developmental Research as a Research Method. In J. Kilpatrick & A. Sierpinska (Eds.), *Mathematics Education as a Research Domain: A Search for Identity (ICMI Study Publication)* (pp. 277-295). Dordrecht, The Netherlands: Kluwer.

Gravemeijer, K. (1999). How emergent models may foster the constitution of formal mathematics. *Mathematical Thinking and Learning*, *1*(2), 155-177.

Martin, J., Oehrtman, M., Roh, C., Swinyard, C., & Hart-Weber, C. (2011). Students' reinvention of definitions of series and pointwise convergence. In S. Brown, S. Larsen, K. Marrongelle, & M. Oehrtman (Eds.), *Proceedings of the Fourteenth Conference on Research in Undergraduate Mathematics Education* (Vol. 1, pp. 239-254), Portland, OR: Portland State University.

Martin, J., Cory, B., Oehrtman, M., & Swinyard, C. (2012). Quantitative reasoning about absolute value during guided reinvention of sequence convergence. Submitted to *The Thirty Forth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, Kalamazoo, MI: University of Western Michigan.

Martin, J., Oehrtman, M., Swinyard, C., & Cory B. (2012). Reinvention six months later: The case of Megan. In S. Brown, S. Larsen, K. Marrongelle, & M. Oehrtman (Eds.), *Proceedings of the The Fifteenth Conference on Research in Undergraduate Mathematics Education* (pp. 109-114), Portland, OR: Portland State University.

Monaghan, J. (1991). Problems with the language of limits. *For the Learning of Mathematics, 11*(3), 20-24.

Oehrtman, M., Swinyard, C., Martin, J., Roh, K., & Hart-Weber, C. (2011). From intuition to rigor: Calculus students' reinvention of the definition of sequence convergence. Paper presented at the 14th Annual Conference on Research in Undergraduate Mathematics Education, Portland, OR: Portland State University.

Roh, K. H. (2008). Students' images and their understanding of definitions of the limit of sequence. *Educational Studies in Mathematics, 69*, 217-233.

Sfard A. (2008). *Thinking as communication*. Cambridge University Press.

Strauss, A., & Corbin, J. (1990). *Basics of qualitative research: Grounded theory procedures and techniques*. Newbury Park, CA: Sage Publications.

Swinyard, C. (2011). Reinventing the formal definition of limit: The case of Amy & Mike. *Journal of Mathematical Behavior, 30*, 93-114.

Swinyard, C., & Larsen, S. (2012). What does it mean to understand the formal definition of limit?: Insights gained from engaging students in reinvention. *Journal for Research in Mathematics Education, 43,* 4, 465-493.

Tall, D. (1992). The transition to advanced mathematical thinking: Functions, limits, infinity, and proof. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 495–511). New York: Macmillan.

Williams, S. R. (1991). Models of limit held by college calculus students. *Journal for Research in Mathematics Education, 22*(3), 219-236.