An Analysis of First Semester Calculus Students' Use of Verbal and Written Language When Describing the Intermediate Value Theorem

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This preliminary report describes the second stage of data collection and analysis in a larger study that examines students' written and verbal language when studying basic theorems in a first-semester calculus course. We examine students' difficulties with understanding and using mathematical language and notation in both formal written work and informal verbal descriptions. Not surprisingly, the students in our study rarely use formal mathematical language without being prompted to do so. One surprising result was that while many students do understand the mathematical notation in the theorems, and can illustrate this graphically when prompted, they still do not use this notation when providing their own written (or verbal) description of a theorem. Preliminary results suggest that our biggest obstacle as teachers is not in getting our students to understand the notation, but instead lies in convincing our students of the power that comes from this notation in describing a concept, thus encouraging our students to use this notation in their own written work.

Keywords: Calculus, Mathematical language, Mathematical notation, Emergent perspective, Constructivism

Introduction and Literature Review

The precision of mathematical language directly contrasts with the imprecise language used daily (both written and spoken) by the average undergraduate student. Additionally, human language is naturally ambiguous and variable, two qualities which only increase students' imprecision. This contrast can become the source of difficulties when these students are expected to read, write, understand and graphically interpret mathematical language and be able to move between these methods of communication freely in an undergraduate mathematics classroom. Our research is motivated by the need to develop a greater awareness of our students' specific understanding and difficulties with mathematical language.

We are particularly interested in how our students understand, interpret and express mathematical theorems given a certain level of conceptual understanding of the content. A significant portion of the recent literature in mathematics education focuses on developing a conceptual understanding of a mathematical entity in the minds of our students. However, in our pilot study, we found that even after students developed a conceptual understanding of a theorem, they were still unable to successfully describe that theorem in a written form. We believe that at least one portion of a solution to this disconnect involves helping students to increase their metalinguistic awareness of their language. The idea of metalinguistic awareness is a term from language and linguistics research and involves developing an ability to objectify and analyze language. It means that students will gain the ability to see the structures of language and manipulate them to achieve the targeted genre of language, specifically mathematical language and notation. Metalinguistic awareness has been widely recognized as an important area of attention for many educational endeavors throughout a student's development, from kindergarten on (Cazden, 1974; González & González, 2000), and we believe that this work will help us to understand how our students come to learn and use the language of mathematics.

The Intermediate Value Theorem (IVT) is often introduced in a first-semester calculus course in the beginning of the semester. As such, it is a convenient theorem in which to begin to understand students' use of mathematical language and notation. In Stewart's *Essential Calculus*, this theorem is introduced in Section 1.5, which discusses an informal notion of continuity. Recall that the IVT states that if a function f is continuous on the closed interval $[a,b]$ and N is any number between $f(a)$ and $f(b)$, where $f(a) \neq f(b)$, then there exists a number c in (a,b) such that $f(c) = N$. (See Figure 1.)

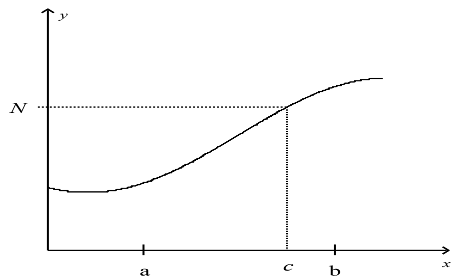

Figure 1: Illustration of the Intermediate Value Theorem

While student understanding of calculus concepts has been investigated enough to result in a relatively large research base (work on limits, functions, derivatives, etc.), the work on theorems is more limited. Abramovitz et al.(2007, 2009) developed a process for *learning* theorems (the self-learning method) to help students better understand the hypotheses and conclusions of the Mean Value Theorem and Rolle's Theorem, though language use was not a focus of this project. There has also been work conducted in the area of how students (and experts) construct and evaluate proofs of theorems in undergraduate mathematics (e.g. Weber, 2001; Weber & Alcock, 2004), though proof of mathematical theorems was not our focus in the current project. Instead, we focus on students' use of mathematical language and their ability to *express* theorems verbally and in writing.

To date, language-related issues in the mathematics education community, including classroom discourse and multi-lingual classrooms, have generated great interest (Brown, 1997; Sfard, 2000; Adler, 2001). However, the intersection of applied linguistics and mathematics education has emerged more recently. The mathematical register is the set of terms and grammatical constructions that are most appropriate for communicating mathematical ideas clearly and with the maximum amount of meaning. Barwell (2005) notes that there has been little attempt to "relate…the acquisition of the mathematical register with the acquisition of mathematical concepts" (p. 97). This work focuses on the development of mathematical language through interacting with mathematics and attention to metalinguistic awareness.

Theoretical Perspective

The theoretical perspective used in this research relies on both on Piaget's structuralism (1970, 1975) as well as on the emergent perspective described by Cobb and Yackel (1996). First, we believe that students must construct for themselves an understanding of the mathematical concepts used in each theorem. As such, the first phase in our classroom teaching of each theorem involves time for the students to explore the concepts, often in the context of an activity designed to elicit specific ideas about each theorem.

Next, we turn to Cobb and Yackel's (1996) emergent perspective to describe the teacher's role in helping the students to understand and use traditional mathematical notation and language. As teachers, we want our students to become comfortable using this formal notation and to see the power this notation provides. Cobb and Yackel (1996) describe the teacher's role as "proactively supporting both students' individual constructions and the evolution of classroom mathematical practices so that students increasingly become able to participate effectively in the mathematical practices of the wider society" (p. 186). Our current research examines which aspects of mathematical language students have appropriated into their own language as well as which aspects they understand, but have not yet taken as part of their own vocabulary.

Phase I: Data & Results

All participants in the study were first-semester calculus students at a large, public research university. The first round of data collection occurred in the Fall 2011 semester in two sections of Calculus I courses, both taught by one of the authors. Two groups of students from each section were videotaped while working on an activity that was designed to guide students to construct an understanding of the hypothesis and conclusion of the Intermediate Value Theorem. This activity was given before the students were formally introduced to the IVT by the instructor. Students were asked to draw a series of functions that satisfied some of the conditions given in the IVT. Two class periods after completing the activity, all students (n = 54) were given a pop quiz which asked students to state the Intermediate Value Theorem in their own words.

Written responses were collected and first analyzed using Corbin and Strauss' (2008) open and axial coding. Preliminary results from this phase were discussed at the 2012 Conference on RUME (Sealey, Deshler, & Toth, 2012). Our leading observation was that students used a series of unconnected indicative clauses to describe some aspects of mathematical theorems without fully understanding all logical possibilities. The data suggested that even though most students were unable to correctly write the theorem, the majority of these students did, in fact, understand the concepts behind the IVT and could express the theorem verbally. The data from the written work certainly showed that students struggled with writing the theorem correctly, but unfortunately, we did not have adequate data to show that these same students could verbally describe the IVT.

Phase II: Data & Results

Phase II of the study was designed to support the hypothesis from Phase I, namely that students are able to verbalize the IVT but not able to accurately express it in written form. Data collection for this phase occurred in the Fall 2012 semester, again in the same author's classroom as the previous year. For this phase, students worked through the same worksheet from Phase I to develop an understanding of the concept of the IVT. During class, the instructor/researcher wrote the theorem on the board and spent a significant portion of class time discussing the mathematical notation used in the theorem. We note that the instruction during this phase was more likely to have addressed many of the issues that students in Phase I were shown to have. Thus, we are not attempting to compare the students from Phase I and Phase II.

Over the next two days, a small group of self-selected students volunteered to participate in out-of-class interviews during which they were asked to describe the IVT in their own words, while being videotaped. After providing a verbal description, they were asked to provide a written description, then watch their previous verbal description (via video-recording) and to compare their written and verbal responses. Finally, the students were asked to draw a graph that illustrated the IVT and discuss how the graph related to their written response.

Video data is currently in the preliminary stages of analysis, though we have preliminary results which contradict our previous hypothesis. Namely, we are not seeing students on video who *can* express the IVT and show an understanding of the concepts behind the IVT, but are unable to express it in written form. Individual interviews seem to indicate that students possess similar written and verbal abilities with respect to being able to describe a mathematical concept. Further analysis is being conducted on the previously collected written work (pop quiz, n=54), which was the basis of Phase I.

Another interesting finding from the preliminary analysis is the discrepancies in what constitutes a "good" statement of a theorem, depending on the mode of language. Specifically, verbal descriptions were initially given higher ratings by the researchers than written descriptions that contained the same mathematical content. While this may not be surprising, it is certainly important to be aware of the discrepancies when evaluating both written and verbal responses from students.

Questions for the Audience

1. Even though our students were knowledgeable about some of the specific mathematical terms used in the formal description of the IVT, they did not provide that information during the task of describing the IVT in their own words. How might we elicit *all* the knowledge the students possess about the mathematical theorem at hand to get a better sense of their full understanding when given a written task, without providing them with prompts to use certain notation?
2. How does this work which appears to show there is not as great a disconnect between the student understanding of a mathematical concept and their ability to express it (either verbally or in written form) as previously thought by researchers fit into the larger mathematics education research knowledge?
3. How do we move forward to understand why there seemed to be a disconnect between written and verbal descriptions in Phase I, but no disconnect in Phase II? We think this could be due to the selection of students (self-selected in Phase II), a result of the instruction (since the instructor/researcher was aware of many issues from Phase I), or possibly simply that talking about the theorem first enabled students to express it in writing in a more coherent way.

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