

Addition and subtraction by human infants

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HUMAN infants can discriminate between different small numbers of items¹⁻⁴, and can determine numerical equivalence across perceptual modalities^{5,6}. This may indicate the possession of true numerical concepts^{1,4-7}. Alternatively, purely perceptual discriminations may underlie these abilities^{8,9}. This debate addresses the nature of subitization, the ability to quantify small numbers of items without conscious counting^{10,11}. Subitization may involve the holistic recognition of canonical perceptual patterns that do not reveal ordinal relationships between the numbers¹², or may instead be an iterative or 'counting' process that specifies these numerical relationships^{4,13}. Here I show that 5-month-old infants can calculate the results of simple arithmetical operations on small numbers of items. This indicates that infants possess true numerical concepts, and suggests that humans are innately endowed with arithmetical abilities. It also suggests that subitization is a process that encodes ordinal information, not a pattern-recognition process yielding non-numerical percepts.

The experiments used a looking-time procedure that has become standard in studies of infant cognition¹⁴⁻¹⁷. Thirty-two infants participated in experiment 1. They were normal, full-term infants with a mean age of 5 months 1 day (range, 4 months 19 days to 5 months 16 days). Infants were divided randomly into two equal groups. Those in the '1+1' group were shown a single item in an empty display area. A small screen then rotated up, hiding the item from view, and the experimenter brought a second identical item into the display area, in clear view of the infant. The experimenter placed the second item out of the infant's sight behind the screen (Fig. 1). Thus, infants could clearly see the nature of the arithmetical operation being perfor-

med, but could not see the result of the operation. The '2-1' group were similarly shown a sequence of events depicting a subtraction of one item from two items (Fig. 1). For both groups of infants, after the above sequence of events was concluded, the screen was rotated downward to reveal either 1 or 2 items in the display case. Infants' looking time to the display was then recorded. Each infant was shown the addition or subtraction 6 times, the result alternating between 1 item and 2 items. Before these test trials, infants were presented with a display containing 1 item and a display containing 2 items and their looking time was recorded, to measure the baseline looking preferences for the two displays.

Infants look longer at unexpected events than expected ones, thus, if they are able to compute the numerical results of these arithmetical operations, they should look longer at the incorrect than at the correct results. The two groups should respond differently to results of 1 and 2 items: the '2-1' group should look longer than the '1+1' group when the result is 2 items than when it is 1 item, which is what is found (Table 1). Pretest trials showed that infants in the two groups did not differ from each other in their baseline looking times to 1 or 2 objects. But in the test trials, infants in the two groups differed significantly—infants in the '1+1' group looked longer at 1, whereas infants in the 2-1 group looked longer at 2. Thus, both groups looked longer at the incorrect than at the correct outcomes (Table 1).

Experiment 2 was a replication of experiment 1 with a smaller number of subjects (sixteen). Their mean age was 4 months 5 days (range, 4 months 18 days to 5 months 5 days). The same pattern of results was obtained; infants in each group looked longer at the incorrect outcome than at the correct outcome (Table 1).

These results show that infants know that an addition or subtraction results in a change in the number of items. But the results are consistent with two distinct hypotheses: (1) that infants are able to compute the precise results of simple additions and subtractions and (2) that infants expect an arithmetical operation to result in a numerical change, but have no expectations about either the size or the direction of the change. They

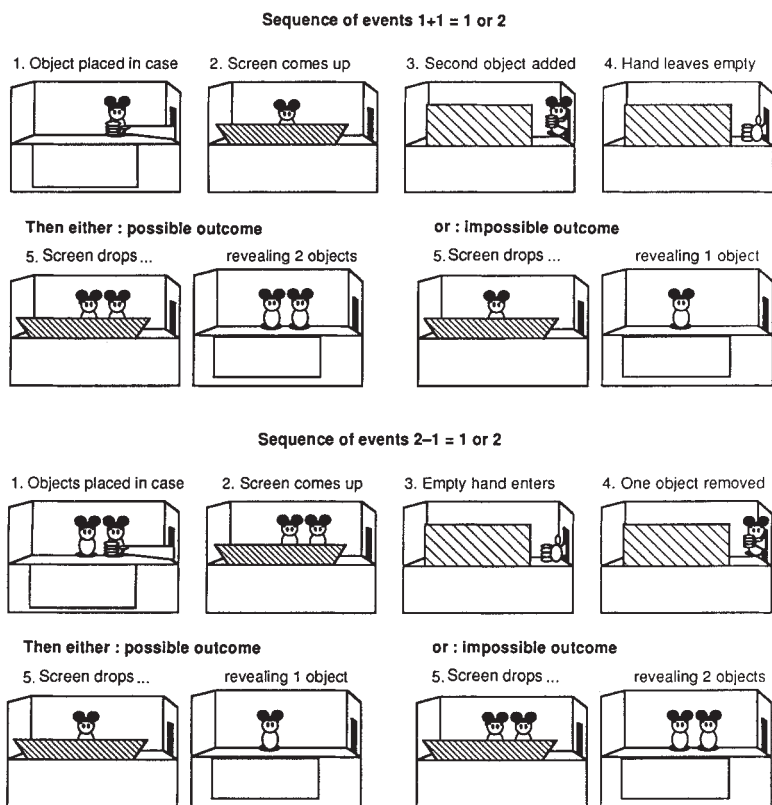


FIG. 1 Sequence of events for '1+1' and '2+1' situations presented in experiments 1 and 2.

METHODS. Trials alternated between a 1-item and a 2-item result, half of the infants received the ordering (1, 2, 1, 2, 1, 2), the remainder receiving the reverse ordering. Infants sat facing the display; parents either stood out of sight behind and not touching the infant, or else gently touched the infant while facing away from the display. The experimenter was hidden behind the display, and manipulated the objects by means of a hidden trap door in the back wall of the display. A hidden observer, unaware of the infant group and of the trial ordering, timed infants' looks to the display. In all experiments, infants were excluded if they became fussy or drowsy during the experiment (16 infants), if their test preference was more than 2.5 standard deviations away from the mean for that group (1 infant), or if they had a pretest preference of more than 10 s for either number (19 infants). The choice of 10 s does not affect the pattern of results (the analyses for experiments 1 and 2 combined give the same pattern even with no cutoff).

TABLE 1 Looking times and preference for 2 items over 1 item

Experiment	Trials	Group	LT(1)*	LT(2)*	P(2)*	d.f.	t	P
1	Pretest	1+1	20.06	20.80	0.74	30	0.649	>0.5
		2-1	17.99	19.61	1.62			
	Test	1+1	13.36	12.80	-0.53	30	2.078	<0.05
		2-1	10.54	13.73	3.19			
2	Pretest	1+1	11.12	10.62	-0.50	14	0.677	>0.5
		2-1	10.35	11.44	1.09			
	Test	1+1	12.08	9.45	-2.65	14	1.795	<0.05
		2-1	10.98	8.05	2.94			
1+2	Pretest	1+1	17.62	18.02	0.41	46	0.873	>0.35
		2-1	15.05	16.47	1.42			
	Test	1+1	13.01	11.89	-1.11	46	2.73	<0.005
		2-1	9.59	12.67	3.09			

Statistical significance was determined by between-group *t*-tests on infants' P(2) values. Probability values are 2-tailed for pretest comparisons, 1-tailed for test comparisons. In experiment 1, a trial concluded when an infant looked away for 2 consecutive seconds after looking at the display for at least 4 cumulative seconds, or had looked for 30 cumulative seconds. Experiment 2; same criteria, except that minimum cumulative looking time was only 2 s. The shorter mean looking times in experiment 2 are probably due to this procedural change. Times are lower in test than pretest trials because infants' looks decrease during the experiment as they become more familiar with the display. Experiment 2, 6 infants in the 1+1 group, 10 infants in the 2-1 group.

* P(2)=LT(2)-LT(1); where P(2), preference for 2; LT(1) and LT(2) are the mean looking times to 1 and 2 items (in seconds).

may simply expect that adding an item to an item will result in some number other than 1; and that subtracting an item from 2 items will result in some number other than 2. To determine whether infants are able to compute the precise results of simple arithmetical operations, I conducted a third experiment.

Experiment 3 tested 16 infants with a mean age of 4 months 18 days (range, 4 months 4 days to 5 months 4 days). Infants were shown a '1+1' addition as before, except that the final number of objects revealed behind the screen was either 2 or 3. In both cases, the result is numerically different from the initial number of items. If infants are computing the exact numerical result of the addition, they would be expected to look longer at the result of 3 items than of 2 items. This pattern was indeed observed (Table 2); infants significantly preferred 3 in the test trials, but not the pretest trials, showing that they were surprised when the addition appeared to result in 3 items. The results from the three experiments support the claim⁷ that 5-month-old human infants are able to calculate the precise results of simple arithmetical operations.

There is an alternative explanation for infants' success in these experiments. Infants may be calculating the results of the addition and subtraction, not of a discrete number of items, but of a continuous amount of physical substance; infants may possess an ability to measure and operate on continuous quantities. But there are reasons to prefer the hypothesis that it is the number of items, not amount of substance, that infants are computing. It has been shown that infants are sensitive to small numerical changes¹⁻⁴, but there is no evidence of a sensitivity to small differences in amount of physical matter. Infants are predisposed to interpret the physical world as composed of discrete, individual entities when perceiving spatial layouts^{14,15}, and they represent the precise spatial locations and trajectories of individual objects relative to each other^{16,17}. Thus, the notion of 'individual entity' plays a prominent role in infants' conceptualization and representation of the physical world, and they

TABLE 2 Looking times and preferences for 3 items over 2

Condition	LT(2)*	LT(3)*	P(3)*	d.f.	t	P
Pretest	14.16	13.87	-0.29	15	-0.224	>0.5
Test	9.96	11.89	1.92	15	2.044	<0.03

Statistical significance was determined by *t*-tests comparing infants' P(3) values to the null hypothesis of no preference. Probability value for pretest comparison is 2-tailed; that for test comparison is 1-tailed. As in experiments 1 and 2, infants were excluded if they showed more than a 10-second pretest preference for one of the numbers; the pattern of results remains the same when these infants are included in the analyses. Experiment 3 used the same criterion for end-of-trial as that used in experiment 2.

* P(3)=LT(3)-LT(2), where P(3), preference for 3; LT(3) and LT(2) are the mean looking times to 3 and 2 items (in seconds).

have abilities that allow them to track distinct entities over time and space. This, together with infants' sensitivity to small numerical differences in collections of items, lends independent support to the hypothesis that infants possess a mechanism for quantifying collections of discrete entities. The most plausible explanation for the findings presented here is that infants can compute the results of simple arithmetical operations.

In sum, infants possess true numerical concepts—they have access to the ordering of and numerical relationships between small numbers, and can manipulate these concepts in numerically meaningful ways. This in turn indicates that the mental process giving rise to these concepts yields true numerical outputs that encode numerical relationships, not holistic percepts derived from a pattern-recognition process. The existence of these arithmetical abilities so early in infancy suggests that humans innately possess the capacity to perform simple arithmetical calculations, which may provide the foundations for the development of further arithmetical knowledge^{7,18}. □

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