Rubik’s Cube and the Mathematics of Other Puzzles

Final Project

Deadline for Project Proposals: Monday, April 11, 2011.

Each student in the class must do an individual final project. A written report on the project is due on the last day of classes, May 6, 2011, unless you are a graduating senior, in which case it is due on Monday, May 2, 2011. In addition to the written report, each student will give a 10 minute oral presentation about their project in one of the last five class meetings.

The project should be about the mathematics involved in a puzzle. The project should describe the solution to the puzzle and the mathematics used in the solution. You may choose any puzzle you wish, or even invent your own puzzle (or a variation on an existing puzzle). You may discuss a puzzle that we have already discussed in class, or one that we have not discussed in class. However, if you choose to focus on a puzzle that we discussed in class, then your report needs to go beyond the analysis that we did in class. (For this reason, it might be a better idea to choose a puzzle that we have not discussed, or a variation on one that we have.)

The project needs to have real, non-trivial, mathematical content. It doesn’t have to be too complicated, but for example, simply describing a puzzle, its history, and its solution without describing any mathematics is not acceptable. Some puzzles do not have much, if any, mathematics associated to them, so they would not be good choices.

The written report must be typed (double spaced) and should be about 5–10 pages long. Because the report will include mathematical content, and because typesetting mathematics is not so easy, figures and formulas can be hand drawn. It might make sense to include such material as appendices. The report should be well-written, clear, fun to read, and explain your analysis of the puzzle in a way that would make sense to any other college student, not just students in the class. You should include historical background about the puzzle if possible.

The oral report is a chance to share your findings with the rest of the class. You may want to use a Power Point presentation, or other on-line materials, but this is not necessary. You may want to pass out copies of complicated diagrams that would be too hard to reproduce on the board in 10 minutes. You may want to have copies of the puzzle with you so that the rest of the class can see the puzzle and try it out, etc. If you give a power point presentation, then you could use a lot of the same materials for your written report, BUT the written report should NOT just be a print out of your power point slides!

The entire project counts for 30% of the course grade. Both the written report and the oral presentation will be graded and count equally. The oral report will be graded primarily on clarity and the extent to which you understand the mathematics behind the puzzle, but attention should also be paid to organization, visual aids, and style. The written report will be graded on clarity (organization, exposition, grammar, spelling, bibliography, etc.), mathematical content (organization, correctness, completeness), and good use of figures, tables, charts, graphs, and so on. The paper must contain a bibliography with all source materials properly cited.
Here are some pointers:

- You MUST understand the mathematics involved in the puzzle. It is better to pick a simple puzzle that you can work out fully and totally understand, than a more difficult puzzle that you read a bunch of stuff about, recite back to me, and don’t really understand. It will be clear to me at your presentation if you really understand it or not.

- Your paper must include the solution to the puzzle. The solution should involve some interesting mathematics!

- DO NOT PLAGIARIZE!. I don’t expect you to invent something new, or discover something new. You can pick a classic puzzle that has already been totally analyzed and learn that analysis and then put it into your paper. But you need to do this without just copying what someone else has written in a book or on the internet. The best way to do this is to read several different sources, work out the solution to your puzzle by yourself, and then write it down in your own words without looking again at the sources. If you want to quote a passage from a source, then do that, but give the proper citation.

- Don’t make this too hard. Keep it simple.

- Do not wait until the last minute! The project is NOT something that you can do in one week. Plan to spend 2 to 3 weeks on your project.

Here are some potential projects:

- Ferrying Puzzles. Look for the history of this class of puzzles. What different examples can you find? Work out the general solution for the Missionary and Cannibals problem when there are \( c \) Cannibals and \( m \) missionaries, with \( c \leq m \). How many trips across the river are needed as a function of \( c \) and \( m \), and so on. Or, solve some other generalization of the classic problem.

- The Knights Tour. There are a lot of results on this problem for various size chess boards. Report on the known results and explain the proofs of some of the more interesting results. Perhaps generalize to your own puzzle on a different shaped boards, etc.

- Permutation Puzzles. There are lots of commercially available puzzles that involve permutations. Often there are plastic numbered pieces that slide around in tracks and that can then be rearranged in different permutations. Report on one of these.

- Puzzles with Pegs. There are lot of puzzles that involve pegs (or marbles) placed in holes on a board then you move the pegs jumping over pieces, etc.

- \( 2 \times 2 \times 2 \) Rubik’s Cube. The classic Rubik’s cube has been recreated in different forms. The simplest is the \( 2 \times 2 \times 2 \) cube. Tell me something interesting about it!

- The Tower of Hanoi.

- Dissection Puzzles. There are lots of puzzles that involve cutting something up into pieces and then putting them back together, possibly in different ways. Tangrams is such a puzzle.
I have many books on puzzles in my office. (Mostly checked out from the library.) If you come to my office hours we can browse the books and I can suggest projects.

Project Proposals

In order to make sure that you have picked a good project, you need to tell me what you plan to do your project on by April 11. Your proposal should be a paragraph or two describing the puzzle and what you plan to say about it. Your proposal should include at least two references that you plan to use for your project.