Math 7 Final Exam Summer, 2005

1. If you roll a pair of dice, what is the chance that:
   (a) you roll doubles?
   (b) both dice are different?
2. If you roll a pair of dice 10 times, what is the chance that:
   (a) you roll *exactly* four pairs of doubles?
   (b) you roll *at least* one pair of sixes?
3. What is the chance of being dealt a full house in poker? That is, you are dealt 5 cards from a full deck and three are of one kind and two are of another kind?
4. A jar contains 4 red marbles, 6 blue marbles and 10 green marbles. If you shut your eyes, reach in and grab 3 marbles, what is the chance that they are all the same color? What is the chance they are all different?
5. Compute the expected value of a 4-number bet in American Roulette. This bet pays 8 to 1. Assume you bet \$1.
6. Compute the expected value of a "hard way 4" bet in craps. This is a multi-roll bet that 4 will come up the "hard way", that is 2–2, before 7 or the "easy way", 1–3. This bet pays 7 to 1. Assume you bet \$1.
7. If you play roulette 300 times, each time betting \$1 on Red, what is the chance that you will finish \$20 or more ahead?
8. Suppose you have a bankroll of \$500 and play \$1 at a time on Don't Pass in craps. (This is an even money bet with the chance of winning equal to 0.493.)
   (a) What is the chance you will reach \$550 *before* going broke?
   (b) If you play until you go broke, about how many plays can you expect to make?
9. A and B play a zero-sum game where the matrix of pay-offs to A is given below.

$$\begin{pmatrix} 3 & -1 & 2 & 0 \\ 0 & 5 & 0 & 6 \\ 1 & 4 & -1 & 5 \end{pmatrix}$$

   Find the optimal strategy for both players and the value of the game to A.
10. BONUS PROBLEM This problem is NOT required. If you do it, you may earn bonus points! Suppose you flip a coin until you get three heads in a row at which point you stop. What is the expected number of coin flips that will take place? Hint: set this up as a random walk on *some* walkway.