Math 177 HW 7 Due Monday, April 2 at 5 pm

44. Given a group G and elements x and y in G, the *commutator of x and y* is $xyx^{-1}y^{-1}$.The commutator $xyx^{-1}y^{-1}$ is sometimes denoted $[x, y]$. The *commutator subgroup* of G, denoted $[G, G]$ is the subgroup generated by all commutators.

    (a) Show that the commutator subgroup of G is normal.

    (b) Show that G is Abelian if and only if the commutator subgroup is trivial.

    (c) Show that $G/[G, G]$ is Abelian.

    (d) Show that the commutator subgroup is the smallest normal subgroup of G such that the quotient of G by this subgroup is Abelian. In other words, show that if N is a normal subgroup of G such that G/N is Abelian, then $[G, G] \subset N$.

45. (Exercise 2, Chapter 4, Johnson) Let $G = \langle X \mid R \rangle$. Show that $G/[G, G]$ has presentation

$$G/[G, G] = \langle X \mid R, [X, X] \rangle,$$

where $[X, X]$ is the set of all relations $[x, y] = 1$ for all $x, y \in X$.

46. (Exercise 3, Chapter 4, Johnson) The *von Dyck* group $D(l, m, n)$ is defined as

$$D(l, m, n) = \langle x, y \mid x^l = 1, y^m = 1, (xy)^n = 1 \rangle.$$

Example 5, Chapter 4 of Johnson, shows that $D(l, m, n) \cong D(n, m, l)$. Show that

$$D(l, m, n) \cong D(m, l, n) \cong D(-l, m, n)$$

and conclude that $D(l, m, n)$ is independent of the order or signs of l, m, and n.

47. (Exercise 4, Chapter 4, Johnson) What group is $D(l, m, n)$ in the following cases?

    (a) One of l, m, n is 1.

    (b) Two of l, m, n are 2.

48. (exercise 10, Chapter 4, Johnson) Show that the group

$$G = \langle a, b, c, d \mid ab = c, bc = d, cd = a, da = b \rangle$$

is cyclic and find its order.

49. (Exercise 17, Chapter 4, Johnson) Let $G = \langle x, y \mid x^2y = y^2x, x^8 = 1 \rangle$. Draw a van Kampen diagram that shows that $y^8 = 1$ in G.