Math 31 HW A Due Tuesday, Feb 27

Write out, in complete sentences, answers to the following questions. Please write clearly!

1. What is a sequence? Give 5 examples.

2. What is a series? Give 5 examples.

3. What is the difference between a sequence and a series?

4. What is the sequence of terms of a series?

5. What is the sequence of partial sums of a series?

6. Give two examples of series and for each one explain what is the associated sequence of terms and associated sequence of partial sums.

7. What does it mean for a sequence to converge? What does it mean for a sequence to diverge? Give two examples of sequences that converge and two that do not.

8. Suppose you are given a sequence and form a new one by deleting the first 100 elements of the first sequence. Explain why the two sequences either both converge (to the same thing) or they both diverge.

9. What does it mean for a series to converge? What does it mean for a series to diverge? Give two examples of series that do not converge because the sequences of partial sums grow without bound (go to infinity).

10. If a series converges, then its sequence of terms must converge to zero. Make a 2 by 2 array with columns labeled "Series Converges" and "Series Diverges" and rows labeled "Sequence of terms converges" and "Sequence of terms diverges". In each row and each column, give an example of series with those properties.

11. Suppose you are given a series and form a new one by throwing away the first 10321 terms of the original series. Explain why either both series converge or they both diverge. Does removing the first 10321 terms of the original series change anything? What?

12. What is a geometric series? Give three examples.

13. Does every geometric series converge? Which ones do? Give three examples of geometric series that converge and explain why. Give three examples of geometric series that diverge and explain why.

14. If a geometric series converges, then there is a simple formula for its sum. What is the formula? Give three examples of convergent geometric series and use the formula to find their sums.

15. What is the harmonic series? Does the sequence of terms of the harmonic series converge? If so, to what? The harmonic series diverges! Why? Give a proof that it diverges.

16. Suppose that $\sum a_i$ and $\sum b_i$ are series and for every i assume that $0 \leq a_i \leq b_i$. What relationship exists between the convergence/divergence of the two series? This is the so-called "Comparison Test". Notice that BOTH series have positive terms!

17. Give 5 examples of nongeometric series that can be shown to be convergent by comparing with a convergent geometric series.

18. Give 5 examples of nongeometric series that can be shown to be divergent by comparing with the harmonic series.

19. What is an alternating series? Give three examples. Does every alternating series converge? Give three examples of alternating series that do not converge.

20. Under what conditions do we know that an alternating series will converge? Give three examples of alternating series that converge and explain why they converge.

21. Explain why the alternating harmonic series converges.

22. If a series is alternating, its terms approach zero, and the magnitude of the terms if strictly decreasing, THEN we know that the difference between any partial sum and the actual sum of the series is smaller than the magnitude of the first neglected term. Give three examples of alternating series and use this fact to estimate the sum of the series to within $1/100$.

23. Give three examples of convergent series for which the 4th term is smaller than $1/100$ but the sum of the first three terms is NOT within $1/100$ of the sum. (Hint: use convergent geometric series for your examples.)