Math 31 HW A Due Tuesday, Feb 27

Write out, in complete sentences, answers to the following questions. Please write clearly!

24. What is the ratio test? Does it apply to ALL series, or just one with positive terms? Give 3 examples of series that can be shown to be convergent by using the ratio test and which are not geometric series.
25. What is integral test? Does it apply to ALL series, or just certain kinds? Give 2 examples of series that can be shown to be convergent using the integral test. Give 2 examples of series that can be shown to be divergent using the integral test.
26. What is a p-series? Are all p-series convergent? If not, how do you know if a p-series is convergent or not?
27. Suppose a convergent series is not geometric and not alternating. How can you determine how many terms are needed to give a partial sum that is within a predetermined amount of the actual sum? Give two examples. In one of the examples, compare the error to a geometric sum. In the other, use an improper integral to bound the error.
28. What is a power series? Give 5 examples.
29. What is the radius of convergence of a power series? How can you find the radius of convergence for a given power series? Give three examples of power series where the radii of convergence are infinite, finite but positive, and zero, respectively.
30. Suppose a power series centered at $x = x_0$ has finite radius of convergence $R > 0$. What can happen at the two points, $x_0 + R$ and $x_0 - R$? Give examples that illustrate all the possibilities.
31. Suppose a power series centered at $x = x_0$ has a positive radius of convergence R. Let $f(x)$ be the function defined by the power series at every point where it converges. How can you get a power series for the derivative, $f'(x)$, on the same interval? What will be the radius of convergence for the power series for $f'(x)$? What happens at the endpoints of the interval of convergence?
32. Give examples of power series with finite radii of convergence that illustrate the following:
    (a) The series converges at both endpoints of the interval of convergence and so does the derivative of the power series.
    (b) The series converges at both endpoints of the interval of convergence but the derivative only converges at the right endpoint of the interval of convergence.

(c) The series converges at both endpoints of the interval of convergence but the derivative only converges at the left endpoint of the interval of convergence.

(d) The series converges at both endpoints of the interval of convergence but the derivative diverges at both endpoints of the interval of convergence.

33. Suppose a power series centered at $x = x_0$ has a positive radius of convergence R. Let $f(x)$ be the function defined by the power series at every point where it converges. How can you get a power series for the antiderivative of $f(x)$? What will be the radius of convergence for the power series for $f'(x)$? What happens at the endpoints of the interval of convergence?

34. Give examples of power series with finite radii of convergence that illustrate the following:

    (a) The series diverges at both endpoints of the interval of convergence and so does the antiderivative of the power series.

    (b) The series diverges at both endpoints of the interval of convergence but the antiderivative converges at the right endpoint of the interval of convergence and diverges at the left endpoint.

    (c) The series diverges at both endpoints of the interval of convergence but the antiderivative converges at the left endpoint of the interval of convergence and diverges at the right endpoint.

    (d) The series diverges at both endpoints of the interval of convergence and so does the antiderivative.