Math 30 Final Exam Study Guide

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The Final Exam will cover Chapters 1-6 of our book. Reread those Chapters and pay special attention to the Chapter Summaries. While not all the exercises were assigned as homework, it would be good to look at ALL the exercises to make sure that you know what they are saying and that you have some idea of how to do them. Of course, many of the exercises in the book involve the use of a computer (primarily to do Euler's method) and we will not be doing these kinds of exercises on the in-class test. The section in Chapter 6 that involves accuracy of a Riemann Sum will not be included on the final exam.

You should specifically know all of the following.

1. You should know and understand the definition of the derivative of a function. Our book defines the derivative of the function $f(x)$ at the point $x = a$ as its rate of change at $x = a$ which is the same as the slope of its graph at the point $(a, f(a))$. But *What is the definition of the slope of the graph at* $(a, f(a))$*?* If you are asked to define the derivative, it will not be sufficient to just say it is the rate of change, or the slope because these three things—derivative, rate of change, and slope—are all synonymous.
2. You should understand the "microscope equation" and be able to use it to estimate values of a function.
3. You should know how to use the microscope equation to estimate absolute or relative errors.
4. Given the graph of a function, you should be able to (roughly) sketch the graph of its derivative.
5. You should know all the differentiation rules and be able to use them to find the derivative of any function.
6. Given a function, you should be able to find its critical points and use this to try and find maxima and minima of the function.

7. You should be able to solve problems where we need to find the maximum or minimum of a function.

8. What does it mean for one quantity to be *proportional* to another?

9. Know what a *linear* function is and how to compute values of a linear function.

10. One of the *the* most important uses of calculus is to *model* real-world problems with differential equations (rate equations). Be familiar with these models:

    (a) The SIR model of the spread of disease.

    (b) Something changing at a rate proportional to itself:

        i. Population growth, or decline

        ii. Radioactive decay

        iii. Newton's Law of Cooling

    (c) Logistic growth

    (d) Predator Prey models

11. One way that can *always* be used to approximate the solution to any model using differential equations is *Euler's method*. Know what this is and how to use it.

12. *Some* differential equations we can solve exactly and therefore do not need to use Euler's method.

13. Given a differential equation, you should be able to decide if a given function is a solution to the differential equation.

14. You should know how to solve the differential equation $y' = ky$ with initial condition $y(0) = C$ and be able to use this in problems related to population growth, radioactive decay, or Newton's Law of Cooling.

15. We *might* be able to solve $y'(x) = f(x)$ IF we can find a function $F(x)$ whose derivative is $f(x)$, in otherwords, if we can antidifferentiate $f(x)$.

16. We discussed a number of settings where we found better and better approximations to something we were trying to calculate. Be familiar with these

    (a) Euler's Method

    (b) Newton's method for solving the equation $f(x) = 0$.

17. Know what a Riemann Sum of a function on some interval is.

18. Know what the Fundamental Theorem of Calculus is.

19. Be able to use the Fundamental Theorem of Calculus to compute integrals.

20. Be able to find the area of simple regions in the plane using integration.