

Final Review

I. Probability

- a. permutations and combinations
- b. conditional probability
- c. the addition rule
- d. the multiplication rule
- e. the subtraction rule
- f. Bayes' theorem

II. Special Distributions: You should be familiar with the following special probability distributions and the relationships between these distributions.

- a. Binomial Distribution
- b. Normal Distribution $N(\mu, \sigma^2)$
- c. Standard Normal Distribution $N(0,1)$
- d. Chi-Square Distribution χ_k^2
- e. Student t Distribution
- f. F Distribution $F(n, m)$
- g. Poisson Distribution
- i. Exponential Distribution

III. Point Estimation

We normally use $\bar{X} = \frac{1}{n} \sum X_i$ as an estimator for the population mean μ and $s_x^2 = \frac{1}{n-1} \sum (X_i - \bar{X})^2$ as an estimator for the population variance σ_x^2 .

III. Confidence Intervals: Be able to derive confidence intervals for the following cases.

a. population mean when the true variance is known

use the fact that $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0,1)$

b. population mean when the true variance is unknown

use the fact that $\frac{\bar{X} - \mu}{s_x/\sqrt{n}} \sim \text{Student } t \text{ with } n-1 \text{ degrees of freedom}$

c. population variance

use the fact that $\frac{(n-1)s_x^2}{\sigma_x^2} \sim \chi_{n-1}^2$

d. population proportion

use the fact that $\hat{\pi} \sim N\left(\pi, \frac{\hat{\pi}(1-\hat{\pi})}{n}\right)$

IV. Hypothesis Testing: Be able to test hypotheses about the following

a. population mean when the true variance is known

use the fact that $\frac{\bar{X}-\mu}{\sigma_x/\sqrt{n}} \sim N(0,1)$

b. population mean when the true variance is unknown

use the fact that $\frac{\bar{X}-\mu}{s_x/\sqrt{n}} \sim \text{Student } t \text{ with } n-1 \text{ degrees of freedom}$

c. population variance

use the fact that $\frac{(n-1)s_x^2}{\sigma_x^2} \sim \chi_{n-1}^2$

d. population proportion

use the fact that that $\hat{\pi} \sim N\left(\pi, \frac{\hat{\pi}(1-\hat{\pi})}{n}\right)$

e. difference between two population means when their variances are known

use the fact that $\frac{(\bar{X}-\bar{Y})-(\mu_x-\mu_y)}{\sqrt{\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}}} \sim N(0,1)$

f. difference between two population means when their variances are unknown but we can assume that the variances are the same.

use the fact that $\frac{(\bar{X}-\bar{Y})-(\mu_x-\mu_y)}{\sqrt{s^2\left(\frac{1}{n_x} + \frac{1}{n_y}\right)}} \sim \text{Student } t \text{ with } n_x + n_y - 2 \text{ degrees of freedom where}$

$$s^2 = \frac{(n_x-1)s_x^2 + (n_y-1)s_y^2}{n_x + n_y - 2}.$$

g. difference between two population means when their variances are unknown but our sample sizes are very, very large.

use the fact that $\frac{(\bar{X}-\bar{Y})-(\mu_x-\mu_y)}{\sqrt{\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}}} \rightarrow N(0,1)$

h. difference between two population means when their variances are unknown and our sample sizes are small.

use the fact that $\frac{(\bar{X}-\bar{Y})-(\mu_x-\mu_y)}{\sqrt{\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}}} \rightarrow \text{Student } t \text{ distribution with the smaller of } (n_x - 1) \text{ and}$

$(n_y - 1) \text{ degrees of freedom (this is an approximation)}$

i. the equality of two population variances

use the fact that $\frac{s_x^2/\sigma_x^2}{s_y^2/\sigma_y^2} \sim F(n_x - 1, n_y - 1).$

j. the equality of two population proportions

use the fact that $\hat{\pi}_1 - \hat{\pi}_2 \sim N\left(\pi_1 - \pi_2, \frac{\pi_1(1-\pi_1)}{n_1} + \frac{\pi_2(1-\pi_2)}{n_2}\right)$

note that if $\pi_1 = \pi_2$, then $\frac{\pi_1(1-\pi_1)}{n_1} + \frac{\pi_2(1-\pi_2)}{n_2} = \pi(1-\pi)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)$ and we use $\hat{\pi} = \frac{X_1+X_2}{n_1+n_2}$

V. Regression: Be able to do hypothesis testing with regression output

For simple hypotheses like $H_0: \beta_2 = 0$, use the fact that $\frac{b-\beta}{SE_b} \sim$ Student t distribution with $N-k$ degrees of freedom.

For hypotheses like $H_0: \beta_2 = \beta_3 = \dots = \beta_k = 0$, use the fact that $\frac{(TSS-RSS)/k-1}{RSS/(N-k)} \sim F(k-1, N-k)$ where k is the number of parameters in the regression equation.