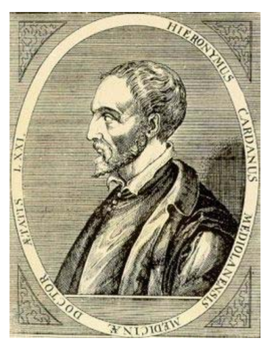
Statistics
Economics 91

Linus Yamane

Probability

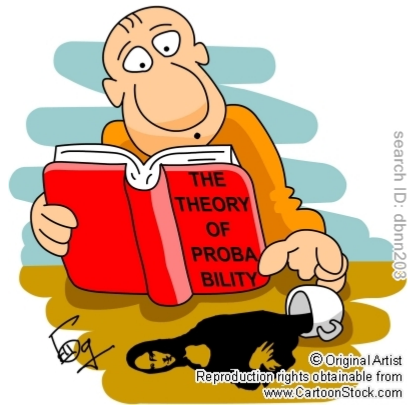
- **Gerolamo Cardano**
  - **1501-1576**
- *Book of Games of Chance*
  - *1526 (published 1663)*



- Gambler, effective cheating methods

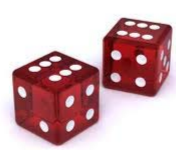
Probability

- Classical probability
- Frequentist probability
- Subjective probability
  - Bayesian probability

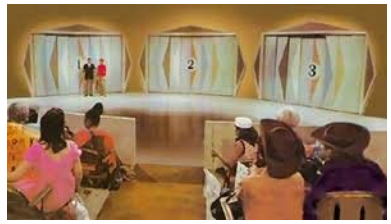


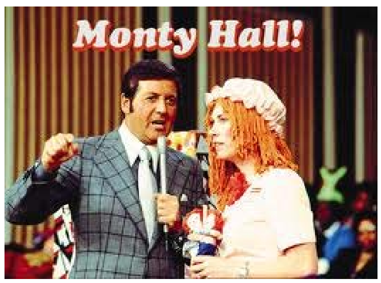
Classical Probability

- The probability of one event occurring out of n possible events, when all n are equally likely, is $1/n$.
- The probability of any one of m events occurring when there are n equally likely possible outcomes is m/n.



Let's Make A Deal





Frequentist Probability

- If in a large number (*n*) of identical situations a certain event occurs *m* times, then its probability is *m/n*.

U.S. Presidential Elections

Political Party	Elections Won
Republican	24
Democratic	22
Democratic-Republican	6
Federalist	2
Whig	2
No Party	1
TOTAL	57

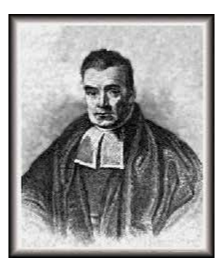
P(Republican) = 24/57 = 42.1%

P(Democrat) = 22/57 = 38.6%

P(Whig) = 2/57 = 3.5%

Bayesian Probability

- An event has subjective probability *m/n* of occurring if you are indifferent between a gamble hinging on this event and a gamble hinging on the draw of a red card from a deck of cards in which a fraction *m/n* of the cards are red.
- Thomas Bayes (1701-1761)



Going back to Classical Probability

- The probability of any one of *m* events occurring when there are *n* equally likely possible outcomes is *m/n*.
- Sometimes it is difficult to count the number of possible outcomes or events.
- We need some tools to help us count.

Standard Counting Procedures

1. Ordering *n* distinct items

-Every possible arrangement of n objects

-The number of different orderings of *n* distinct items is n factoral:

$$n! = n(n-1)(n-2)\cdots 1$$

Standard Counting Procedures

2. Permutations of *x* out of *n* distinct items

The number of different permutations of *x* out of *n* distinct items is

$$P_x^n = {}_nP_x = n(n-1)(n-2)\ldots(n-x+1) = \frac{n!}{(n-x)!}$$

Order matters here

Standard Counting Procedures

3. Combinations of *x* out of *n* distinct items

The number of *x*-item combinations using *n* distinct items

$$C_x^n = {}_nC_x = \binom{n}{x} = \frac{n!}{x!\,(n-x)!}$$

Note: 0! = 1 and $C_0^n = C_n^n = 1$

Order does not matter here

California Lottery

Pick 5 numbers (1-70) and
1 mega number (1-25)

Pick 5 numbers (1-47) and 1
mega number (1-27)

Pick 5 numbers (1-39)

Probability Definitions

- P(A) = P[A] = probability of event A
- P[A∩B] = P[A and B] = probability of all outcomes that belong to A and B
- P[AUB] = P[A or B] = probability of all outcomes that belong to A or B
- P[A|B] = conditional probability of A given B
- P[A|B] = P[A∩B]/P[B]
- P[$\bar{A}$] = probability of not A (complement of A)

Properties

- If A and B are **independent**, then P[A|B] = P[A] and P[B|A] = P[B]
  - If A is independent of B, B is independent of A
- A and B are **mutually exclusive** if A∩B=∅.
  - The intersection of A and B is the empty set
  - P[A∩B] = 0
- If A and B are *independent*, A and B are NOT *mutually exclusive*

Rules of Probability

Addition Rule

P[AUB] = P[A] + P[B] – P[A∩B]

Multiplication Rule

P[A∩B] = P[A]P[B|A] = P[B]P[A|B]

Subtraction Rule

P[A] = 1 - P[not A] = 1 – P[$\bar{A}$]

Bayes Theorem

$$P[B|A] = \frac{P[A \cap B]}{P[A]} = \frac{P[A|B]P[B]}{P[A]}$$

$$P[B|A] = \frac{P[A|B]P[B]}{P[A|B]P[B] + P[A|\bar{B}]P[\bar{B}]}$$